Wolff-Denjoy theorems in non-smooth convex domains

Marco Abate
Università di Pisa, Italy
e-mail: abate@dm.unipi.it

In this talk, we shall describe new proofs (due to Budzynska-Kuczumow-Reich and Abate-Raissy) of a Wolff-Denjoy theorem in not necessarily smooth strongly convex domains, extending to the non-smooth case the description of the dynamics of holomorphic self-maps known for smooth strongly convex domains. We shall also present some extensions to (not necessarily smooth) weakly convex domains. (Joint work with J. Raissy, Université Paul Sabatier, Toulouse)

Viscous fingering in the evaporation fronts of thin liquid films

Oded Agam
The Hebrew University of Jerusalem, The Racah Institute of Physics, Israel
e-mail: agam@phys.huji.ac.il

A thin water film on a cleaved mica substrate undergoes a first-order phase transition between two values of film thickness. By inducing a finite evaporation rate of the water, the interface between the two phases develops a fingering instability similar to that observed in the Saffman-Taylor problem. The dynamics of the droplet interface is dictated by an infinite number of conserved quantities: all harmonic moments decay exponentially at the same rate. Using this property, we construct solutions describing the dynamics of evaporation of simply connected droplets as well as the nucleation of a dry patch within the droplet domain. We also consider a new type of localized Rayleigh instability, which is developed during evaporation along the rim of the droplet.

A univalence criterion and its application to the error function

Dov Aharonov
The Technion - Israel Institute of Technology, Israel
e-mail: dova@techunix.technion.ac.il

We generate a (bi-parametric) family of sufficient conditions for the univalence of analytic functions in the unit disk. These criteria may be applied, for example, to estimate the radius of univalence of the error function $\mathrm{errf}(z)$. (Joint work with Uri Elias, The Technion.)

The separation of singularities of holomorphic functions

Lev Aizenberg
Bar-Ilan University, Israel
e-mail: aizenbrg@gmail.com

A statement is proved on the separation of singularities of holomorphic functions from the H^p, $p > 1$, spaces over strictly pseudoconvex domains.

Reconstructing a function from its V-line averages in a disc

Gaik Ambartsoumian
The University of Texas at Arlington, USA
e-mail: gambarts@uta.edu

Several novel imaging modalities proposed during the last couple of years are based on a mathematical model, that uses the V-line Radon transform (VRT). This transform, sometimes also called broken-ray Radon transform, integrates a function along V-shaped piecewise linear trajectories composed of two intervals in the plane with a common endpoint. Image reconstruction problems in these modalities require inversion of the VRT.

In this work, we present two different exact methods of inverting the VRT of functions supported in a disc of arbitrary radius. In both cases, we use a two-dimensional restriction of VRT data, with the incident rays normal to the boundary of the disc and a fixed breaking angle. The first inversion formula employs a connection of the ordinary Radon transform and VRT in the circular geometry of data acquisition and requires a full data set. The second one is based on Fourier expansion techniques, and can be used in certain cases of image reconstruction from incomplete data.

Accurate Fourier reconstruction of piecewise-smooth functions

Dmitry Batenkov
Weizmann Institute of Science, Rehovot, Israel
e-mail: dima.batenkov@gmail.com

Classical approximation theory establishes the link between the regularity of a function $f : [-\pi, \pi] \to \mathbb{R}$ and the convergence rate of its approximation by partial Fourier sums $\mathcal{F}_n = \sum_{|k|\le n} c_k e^{\imath k x}$, where $c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\, e^{-\imath k t} dt$. In particular, if $f \in C^d$ is 2π-periodic, then $|f - \mathcal{F}_n| \sim n^{-d-1}$ uniformly. However, if f is discontinuous only at a finite number of points, there is no uniform convergence (due to the Gibbs phenomenon), while away from discontinuities the convergence rate drops to n^{-1}. Several alternative summation methods are well-known, which can restore the fast convergence up to the discontinuities, but only if their locations are known in advance. During the last decades, the problem of accurate reconstruction of the discontinuity locations has received much attention. In particular, it has been conjectured that they can be restored with the "classical" (the best possible) rate n^{-d-2} from the Fourier coefficients by some nonlinear method. Recently we have settled the above conjecture by devising an approximation procedure based on the so-called Eckhoff's method and proved that it recovers the jumps with the required accuracy.

Automorphic-invariant isometric operators and their unitary extensions

Miron Bekker
University of Pittsburgh at Johnstown, USA
e-mail: bekker@pitt.edu

We consider isometric operators with finite defect numbers that are unitarily equivalent to their Möbius transformations. A functional characterization of such isometries is given in terms of their characteristic operator functions. We show that any such isometry admits a contractive extension that is also unitarily equivalent to its Möbius transformation and a unitary extension in a larger space that has the same property. Examples of automorphic invariant operators are considered.

Linear instability of the relativistic Vlasov–Maxwell system

Jonathan Ben-Artzi
University of Cambridge, UK
e-mail: J.Ben-Artzi@damtp.cam.ac.uk

We consider the relativistic Vlasov–Maxwell system of equations which describes the evolution of a collisionless charged plasma. We show that under rather general conditions, one can test for linear instability by checking the spectral properties of Schrödinger-type operators that act only on the spatial variable, not the full phase space. This extends previous results that show linear and nonlinear stability and instability in more restrictive settings.

Cyclicity in the Dirichlet spaces and extremal polynomials

Catherine Beneteau
University of South Florida, USA
e-mail: cbenetea@usf.edu

In this talk, I will consider functions that belong to the Dirichlet space, the space of analytic functions in the disk whose derivatives are square area integrable. In particular, I will examine the cyclic functions in this space, that is the functions whose polynomial multiples generate the whole space. I will discuss the question of how to construct the polynomials p_n such that $p_n f$ approach 1 in the Dirichlet norm and will give sharp estimates on the rate of decay of the norm of $p_n f - 1$, in particular for functions f that have zeros on the circle. I will also examine some easier variants of the Brown and Shields Conjecture. This work is joint with Alberto Condori, Constance Liaw, Daniel Seco, and Alan Sola.

Shift-invariant subspaces, inner functions and related linear systems: the weighted Bergman space setting

Vladimir Bolotnikov
College of William and Mary, Virginia, USA
e-mail: vladibo62@gmail.com

Subspaces of the Hardy space of the unit disk which are invariant under the backward shift appear as the ranges of observability operators associated with a discrete-time-invariant linear system as well as the functional-model space for a Hilbert space contraction, while forward shift-invariant subspaces admit representations in terms of inner functions which can be written in terms of transfer-function realizations. We will discuss an analogue of these ideas in the context of weighted Hardy spaces over the unit disk.

The Julia-Wolff-Carathéodory theorem(s) in higher dimensions for mappings and infinitesimal generators

Filippo Bracci
Università di Roma "Tor Vergata", Italy
e-mail: fbracci@mat.uniroma2.it

I will talk about new results on the boundary behaviours of mappings and infinitesimal generators at regular boundary points.

Affine modulus and Nitsche type problem

Daoud Bshouty
The Technion - Israel Institute of Technology, Israel
e-mail: daoud@technion.ac.il

Nitsche's Conjecture proved recently by Iwaniec, Kovalev and Innonen [1] states

A harmonic homeomorphism from the annulus $\{z : 1 < |z| < r\}$ *to an annulus* $\{w : 1 < |w| < R\}$ *if, and only if* $R \geq \frac{1}{2}(r + \frac{1}{r})$.

Having in mind that a conformal mapping (analytic homeomorphism) between any two doubly connected domains exists if and only if they have identical modulus, it seems reasonable to reformulate the Nitshe's bound in terms of moduli. This is discussed in a subsequent paper of the authors [2].

Harmonicity is preserved under a conformal change of the variable z and an affine transformation of the variable w. The modulus of a doubly connected domain is invariant under a conformal change of the variable yet this is not the case under an affine transformation. We therefore define the *affine modulus* of a doubly connected domain Ω to be the supremum of the moduli of $\phi(\Omega)$ over all affine mappings ϕ. The affine modulus of Ω is denoted by $Mod_{@}\Omega$.

The interpretation of the Nitsche bound in terms of the moduli becomes

A harmonic homeomorphism f *from* D *onto* $\{w : 1 < |w| < R\}$ *exists if, and only if,* $Mod_{@}\Omega \geq \log(\cosh\ Mod\ D)$.

In this respect, the authors conjecture that this is the best possible bound for arbitrary target domain and pose the problem to test the cases where the target is a Teichmüller or Grötsch domain. In this talk, we shall consider the first case.

References

[1] T. Iwaniec, L. V. Kovalev, and J. Onninen, The Nitsche conjecture, *J. Amer. Math. Soc.,* **24**(2) (2011), 345-373.

[2] T. Iwaniec, L. V. Kovalev, and J. Onninen, The harmonic mapping problem and affine capacity, *Proc. Roy. Soc. Edinburgh Sect. A* **141**(5)(2011), 10171030.

The Denjoy–Wolff Theorem in complex Banach spaces

Monika Budzyńska
Uniwersytet Marii Curie-Skłodowskiej w Lublinie, Poland
e-mail: monikab1@hektor.umcs.lublin.pl

If D is a bounded and strictly convex domain in a complex Banach space and $f : D \to D$ is holomorphic, condensing with respect to the Kuratowski measure of noncompactness and fixed-point-free, then there exists $\xi \in \partial D$ such that the sequence $\{f^n\}$ of the iterates of f converges in the compact-open topology to the constant mapping taking the value ξ.

Uniqueness of photon spheres in static spacetimes

Carla Cederbaum
Duke University, NC, US, Tübingen University, Germany
e-mail: carla@math.duke.edu

It is well known that any vacuum static spacetime possessing a black hole horizon must be a Schwarzschild solution. We will present a proof that the same result carries over to vacuum static spacetimes possessing a photon sphere. The proof is similar to W. Israel's original proof of static black hole uniqueness.

The Essential Spectrum of the Laplacian on complete manifolds

Nelia Charalambous
University of Cyprus, Cyprus
e-mail: nelia@ucy.ac.cy

The essential spectrum of the Laplacian on functions has been extensively studied. It is known that on hyperbolic space a spectral gap appears, whereas it has been conjectured that on manifolds with uniformly subexponential volume growth and Ricci curvature bounded below the essential spectrum is the nonnegative real line. Our goal was to generalize the set of manifolds on which the essential spectrum is the nonnegative real line. In our work with Zhiqin, Lu we prove a generalization of Weyl's criterion for the essential spectrum. We then apply this generalized criterion to prove that on manifolds with Ricci curvature

asymptotically nonnegative in the radial direction the essential spectrum is the nonnegative real line. Our condition on curvature only imposes subexponential volume growth at a point and thus significantly improves upon previous results. We also use our criterion to compute the essential spectrum of a complete shrinking Ricci soliton and study the essential spectrum of the Laplacian on forms.

Well-Posedness for Degenerate Schrödinger Equations

Massimo Cicognani
Università di Bologna, Italy
e-mail: massimo.cicognani@unibo.it

We consider the initial value problem for Schrödinger type equations

$$\frac{1}{i}\partial_t u - a(t)\Delta_x u + \sum_{j=1}^{n} b_j(t,x)\partial_{x_j} u = 0$$

with $a(t)$ vanishing of finite order at $t = 0$ proving the well-posedness in Sobolev and Gevrey spaces according to the behavior of the real parts $\operatorname{Re} b_j(t,x)$ as $t \to 0$ and $|x| \to \infty$. Moreover, we discuss the application of our approach to the case of a general degeneracy.

Boundary behavior of the iterates of a self-map of the unit disk

Manuel D. Contreras
Universidad de Sevilla, Spain
e-mail: contreras@us.es

In this talk we will discuss a proper boundary Denjoy–Wolff theorem. Mainly, we will focus in parabolic self-maps of the unit disk of zero hyperbolic step whose Koenigs function has an angular limit almost everywhere on the boundary of the unit disk. We also provide some quantitative information about this convergence. This is a joint work with S. Díaz-Madrigal and Ch. Pommerenke.

Effectiveness of a scale-invariant damping for semilinear waves

Marcello D'Abbicco
University of Brescia, Italy
e-mail: m.dabbicco@gmail.com

We prove the global existence of the small data solution to

$$u_{tt} - \triangle u + \mu(1+t)^{-1} u_t = |u|^p\,, \quad u(0,x) = u_0(x)\,, \quad u_t(0,x) = u_1(x)\,,$$

in space dimension $n = 1, 2$. We assume the data to be small in the energy space $H^1 \times L^2$ and possibly in L^1. Thanks to the additional regularity L^1, we obtain global existence for $p > 1 + 2/n$, provided that $\mu \geq n + 2$. Without the additional L^1 regularity we get global existence for $p > 1 + 4/n$, provided that $\mu \geq 2$. These two values of μ correspond to the

thresholds which guarantee that the damping is *effective* with respect to the $(L^1 \cap L^2) - L^2$ and $L^2 - L^2$ decay estimates for the linear equation. By *effectiveness*, we mean that the obtained decay rates are the same as the corresponding decay estimates for the linear heat equation

$$-\triangle v + \mu(1+t)^{-1} v_t = 0\,, \quad v(0,x) = v_0(x)\,.$$

References

[1] M. D'Abbicco, The Threshold between Effective and Noneffective Damping for Semilinear Waves, arXiv:1211.0731 [math.AP].

[2] J. Wirth, Solution representations for a wave equation with weak dissipation, *Math. Meth. Appl. Sci.* **27** (2004), 101–124, doi: 10.1002/mma.446.

H^p Spaces of s-regular functions

Chiara de Fabritiis
Università Politecnica delle Marche Via Brecce, Italy
e-mail: fabritiis@dipmat.univpm.it

The notion of s-regular function of a quaternionic variable was introduced some years ago by Gentili and Struppa in order to generalize holomorphic functions. We define H^p, the analogous of Hardy spaces on the disk, in the setting of slice-regular fuctions and investigate on the structure of these spaces, proving several results on their behaviour.

Local and global aspects in Loewner theory

Santiago Díaz-Madrigal
Universidad de Sevilla, Spain
e-mail: madrigal@us.es

We expose some recent results concerning similarities and differences for decreasing and increasing Loewner chains. We pay special attention to the very classical cases.

A new version of circular symmetrization with applications to the geometric function theory

Vladimir Dubinin
Far-Eastern Branch of the Russian Academy of Science, Russia
e-mail: dubinin@iam.dvo.ru

A new version of circular symmetrization of sets, functions and condensers is proposed, which is different from the classical circular symmetrization of Polya in the following respect: the symmetrized sets and condensers lie on the Riemann surface of the inverse function of a Chebyshev polynomial. As applications of the new symmetrization principle, some new theorems in the geometric function theory are proved. For example, Hayman's well-known results for nonvanishing p-valent holomorphic functions are supplemented as well as results

for p-valent functions in a disc, which have a zero of order p at the origin [1].The well-known extremal problems of Grotzsch and Teichmuller for the moduli of plane doubly connected domains are extended to the case of domains on Riemann surfaces [2]. In this lecture, we focus on the applications for the complex polynomials. In particular, we obtain a precise version of the Markov-type inequality for arbitrary compact sets in the complex plane that take into account the critical values of polynomials. Also, an exact lower bound is established for maximal moduli of the critical values of polynomials P of degree n with fixed first and leading coefficients and normalized by $P(0) = 0$ [3].

References

[1] V. N. Dubinin, A new version of circular symmetrization with applications to p-valent functions, *Sbornik: Mathematics* **203:7** (2012), 996–1011.

[2] V. N. Dubinin, The Grötzsch and Teichmüller extremal problems on a Riemann surface, *Mathematical Notes* **96:2** (2012), 773–780.

[3] V. N. Dubinin, On an extremal problem for complex polynomials with constraints on their critical values, *Sbornik: Mathematics*, to appear.

A reverse Schwarz–Pick inequality

Konstantin Dyakonov
ICREA and Universitat de Barcelona, Spain
e-mail: konstantin.dyakonov@icrea.cat

We prove a kind of "reverse Schwarz-Pick lemma" for holomorphic self-maps of the disk. The result becomes especially clear-cut for inner functions and casts new light on their derivatives.

Analytic functions associated with functions of bounded variation

Jacek Dziok
Uniwersytet Rzeszowski, Poland
e-mail: jdziok@univ.rzeszow.pl

The object of the talk is to investigate classes of functions associated with functions of bounded rotation. We obtain characterizations and various inclusion relationships between defined classes of functions. Some applications of the main results are also considered.

Nonlinear stability for the Einstein-Vlasov system

David Fajman
Universität Wien, Austria
e-mail: david.fajman@aei.mpg.de

Solutions to the Einstein-Vlasov system describe spacetimes with collisionless matter. The nonlinear stability problem for the Einstein-Vlasov system with symmetries has been

considered in a series of works starting with Rein and Rendall in 1992. Recently, the first result for the Einstein-Vlasov system without symmetry assumptions has been established by Ringström, considering a positive cosmological constant. In the talk, we present the proof of future nonlinear stability of the Einstein-Vlasov system in 2+1 dimensions without symmetry assumptions and cosmological constant. Due to the slow expansion and low spatial dimension in that situation, it is essential to prove strong decay properties of the energy momentum tensor. We obtain these decay rates, by introducing geometric Vlasov energies using a specific metric on the tangent bundle of spacelike hypersurfaces - the Sasaki metric. We present energy estimates for those energies and their application in the proof of nonlinear stability. Finally, we give an outlook to applications and related work in progress on the corresponding higher dimensional problem.

On large time decay of solutions to equations of Korteweg-de Vries type

Andrei Faminskii
Peoples' Friendship University of Russia, Russia
e-mail: afaminskii@sci.pfu.edu.ru

Decay for large time of solutions to generalized Korteweg-de Vries equation with certain damping is studied. The initial value problem and initial-boundary value problems on a half-line and a bounded interval are considered. Certain sufficient assumptions providing such a decay are established. The results are extended to multi-dimensional generalizations such as Zakharov-Kuznetsov equation.

Causal mathematics in dynamical systems

Efim Galperin
University of Quebec at Montreal, Canada
e-mail: galperin.efim@uqam.ca

The classical Newton-Leibniz right time derivatives are noncausal and physically inexistent at the current time of their definition. Their formal application to dynamical systems is incorrect, physically invalid and has already led to catastrophic situations in autopilot systems in the case of failure of the outboard Pitot tubes. The new concept of causal higher order derivatives will be presented in the generalized law of motion for application to the causal dynamical systems in engineering and technology.

Microlocal Analysis for Wave Equation in the Einstein & de Sitter Spacetime

Anahit Galstyan
University of Texas-Pan American, USA
e-mail: agalstyan@utpa.edu

The talk is concerned with the waves propagating in the universe modeled by the so-called Einstein–de Sitter cosmological model. The wave equation in the Einstein & de Sitter Spacetime is strictly hyperbolic in the domain with positive time, while on the initial hyper

surface its coefficients have singularities. That makes difficult the study of the initial value problem. In particular, the Cauchy problem for the wave equation in the Einstein–de Sitter spacetime is not well-posed. We introduce the initial value problem for this equation and give the parametrixes in the terms of Fourier integral operators. The estimates for the energy of solutions will be presented as well.

Regular functions of a quaternionic variable and orthogonal complex structures

Graziano Gentili
Università di Firenze, Italy
e-mail: gentili@math.unifi.it

In this talk, the recent theory of slice regular functions of a quaternionic variable will be briefly presented. Applications to the classification of orthogonal complex structures will be illustrated.

The singular perturbation problem for Kirchhoff equation: sharp decay-error estimates

Marina Ghisi
Università di Pisa, Italy
e-mail: ghisi@dm.unipi.it

We consider a family of Kirchhoff equations with a small parameter in front of the second-order time-derivative and a dissipation term. We prove optimal decay estimates for the hyperbolic problem and optimal decay-error estimates for the difference between solutions of the hyperbolic and the parabolic problem. These estimates show a quite surprising fact: in the nondegenerate case, the analogy between parabolic equations and dissipative hyperbolic equations is weaker than in the degenerate case. In the degenerate case under some assumption on the initial data, the difference between solutions decays faster than the two terms separately while under the complementary assumption the optimal decay-error estimates involve a decay rate that is slower than the decay rate of the two terms.

Some metric properties of polynomials on compact homogeneous spaces

Victor Gichev
Omsk Branch of Sobolev Institute of Mathematics, Russia
e-mail: gichev@ofim.oscsbras.ru

Let $M = G/H$ be a homogeneous space of a compact Lie group G. We say that u is a polynomial on M if it belongs to some G-invariant finite dimensional subspace $\mathcal{E}$ of $L^2(M)$. If M is Riemannian and G acts by isometries, then the polynomials are finite sums of the Laplace–Beltrami eigenfunctions. Set $\alpha_a(u) = \int_M \left(u_+^a(x) - u_-^a(x)\right) dx$, where $u_+(x) = \max\{u(x), 0\}$, $u_-(x) = -\min\{u(x), 0\}$. We consider the asymptotic behavior of the variance of the random variable α_a as $a \to \infty$ or $\dim \mathcal{E} \to \infty$ for the uniform distribution

of u on the unit sphere $\mathcal{S}$ in $\mathcal{E}$, which can be treated as a measure of asymmetry of u in the mean. For $a = 0$, α_a is the difference between the measures of positivity and negativity of u. Marinucci and Wigman proved for the space $\mathcal{E} = \mathcal{H}_n$ of the spherical harmonics of degree n on S^2 that its variance is equivalent to $\frac{C}{n^2}$. In contrast to the expectations of various metric quantities, $\operatorname{Var} \alpha_a$ depends heavily on the spectrum of $\mathcal{E}$. For example, if $\mathcal{E}$ is the space of all trigonometric polynomials of degree n, then it is $\frac{A}{n}(1 + o(1))$ but for the theta sums it is $O\left(n^{-2}\right)$. Also, some estimates of Hausdorff measures of the intersections of level sets and their expectations for some special types of homogeneous spaces will be presented in this talk.

Complex analysis and separation of spectums for ultrahyperbolic differential operators

Simon Gindikin
Rutgers University, USA
e-mail: gindikin@math.rutgers.edu

Let us consider Laplace-Beltrami operator on the hyperboloid of one sheet. How to separate its continuous and discrete spectrums? It turns out that a function lies at the subspace of the continuous spectrum if and only it can be holomorphically extended in some tube at the complexified hyperboloid. Correspondingly, functions of the subspace of the discrete spectrum admit extensions in some (non convex) tube as cohomology of higher degree. There are interesting explicit formulas around of this phenomenon: Fourier coefficients, projections on spectrums.

Dynamical systems with stochastic perturbations in terms of mean derivatives

Yuri E. Gliklikh
Voronezh State University, Russia
e-mail: ygliklikh@gmail.com

The notion of mean derivatives was introduced by E. Nelson in 1960's for the needs of the so-called Nelson's stochastic mechanics (a version of quantum mechanics). Later a lot of other applications of equations with mean derivatives to some branches of science were found. The inclusions with mean derivatives arise in problems with control or in the case of motion in complicated media. The classical Nelson mean derivatives (forward, backward, symmetric, etc.) give information on the drift of a stochastic process. Later by a slight modification of some Nelson's idea, we introduced a new mean derivative, called quadratic, that was responsible for the diffusion coefficient of the process. After that, it became in principle possible to recover a process from its mean derivatives. Note that the natural analogue of the physical velocity of deterministic processes is the so-called current velocity (symmetric mean derivative). It also should be pointed out that different versions of second order mean derivatives are involved into description of various processes of mathematical physics. In the talk, we present a brief introduction into the theory of equations and inclusions with mean derivatives as well as the following new applications:

- Mechanical systems with random perturbations and with control.

• Description of motion of deterministic viscous fluid via equations with mean derivatives on the groups of diffeomorphisms.

• Description of motion of a quantum particle in the classical gauge field in the language of stochastic mechanics.

• Existence of solutions of equations and inclusions with current velocities.

• Description of measurement of dynamically distorted signals with noise.

• Optimal control of equations with mean derivatives (in particular, optimal control of a portfolio with several assets).

Boundary values of holomorphic functions in terms of the argument principle

Josip Globevnik
Univerzi v Ljubljani, Slovenia
e-mail: josip.globevnik@fmf.uni-lj.si

In 2003, the speaker proved the following result about characterizing the boundary values of functions from the disc algebra in terms of the argument principle: Let U be the unit disc in $\mathbb{C}$. A continuous function f on bU extends holomorphically through U if and only if for each polynomial P such that $f+P$ has no zero on bU, the change of argument of $f+P$ along bU is nonnegative. In the talk, we describe some results on characterizing boundary values of holomorphic and meromorphic functions in terms of the argument principle obtained since then and mention some open problems.

Optimal decay estimates for semi-linear parabolic and hyperbolic equations

Massimo Gobbino
Università di Pisa, Italy
e-mail: ghisi@dm.unipi.it

We consider semi-linear parabolic equations and semi-linear dissipative hyperbolic equations. We prove optimal decay estimates for the solutions under suitable assumptions concerning the behavior of the non-linear terms near the origin.

In the case where the linear part has a nontrivial kernel, we show the coexistence of slow solution (with polynomial decay rate) and fast solution (with exponential decay rate). We also classify all possible exponential decay rates.

Singularities of mappings with integrally bounded distortions

Anatoly Golberg
Holon Institute of Technology, Iarael
e-mail: golberga@hit.ac.il

We consider general discrete open mappings in $\mathbb{R}^n$ under which the perturbation of extremal lengths of curve collections is controlled integrally via $\int Q(x)\eta^p(|x-x_0|)dm(x)$, with $n-1<p<n$, where Q is a measurable function on $\mathbb{R}^n$ and $\int_{r_1}^{r_2} \eta(r)dr \geq 1$ for any η on a given interval $[r_1, r_2]$.

The main result is that in contrast to the canonical classical case of holomorphic functions, the above mappings may only have either removable singularities or poles, but never essential singularities. This is a joint work with R. Salimov and E. Sevost'yanov.

Conformal weights and embedding operators

Vladimir Gol'dshtein
Ben Gurion University of the Negev, Israel
e-mail: vladimir@bgu.ac.il

With help of Riemann Mappings Theorem, we introduce a conformal weight h for Lebesgue spaces in any simply connected plane domain Ω with nonempty boundary. The weight h is equal to the Jacobian of a conformal map φ from Ω to the unit disc. The weighed Lebesgue spaces for different conformal weights are isomorphic. It is the reason why we call such weights "conformal weights". We study embedding operators of Sobolev spaces $\overset{\circ}{W}{}^1_p(\Omega)$ into the weighted Lebesgue spaces $L_q(\Omega, h)$. In the case $p = 2$, such embeddings are compact for any $1 \le q < \infty$. For $p \ne 2$, we use Brennan's conjecture results to estimate q.

Non-uniqueness of cycles in some simple non-linear dynamical systems

Vladimir P. Golubyatnikov
Sobolev Institute of mathematics SB RAS, Novosibirsk, Russia
e-mail: glbtn@math.nsc.ru

Classification of periodic trajectories in phase portraits of dynamical systems appear in various domains of mathematics and in applications: in Geometry (closed geodesics on a Riemannian manifold, estimates of their quantities), in Celestial Mechanics, in Ergodic Theory, in modeling of biochemical processes, etc.

In cooperation with biologists, grant 12-01-00074 of RFBR, we study periodic trajectories of odd-dimensional nonlinear dissipative dynamical systems of (bio)-chemical kinetics considered as models of gene networks functioning. These systems can be written in the following form:

$$\frac{dx_1}{dt} = f_1(x_{2k+1}) - x_1; \quad \frac{dx_i}{dt} = f_i(x_{i-1}) - x_i; \quad i = 2, \dots, 2k+1. \tag{1}$$

All the functions f_i here are monotonically decreasing, and the variables $x_i(t) \ge 0$ denote concentrations of $2k + 1$ substances in the gene network (proteins, RNA etc). The first summands in equations (1) describe synthesis of these substances, while the negative terms correspond to their natural degradation.

For these dynamical systems, we construct invariant domains near their stationary points and show that:

1. Each system of the type (1) has a unique stationary point.

2. If this point is hyperbolic, then some its invariant domain Q contains at least one cycle of this system.

This domain Q is non-convex and can be represented as a union (a chain) of $(4k+2)$ triangle prisms $\Delta^2 \times I^{2k-1}$ = 2-dimensional triangle $\times$ $(2k-1)$-dimensional parallelepiped. All trajectories of the system (1) which start in Q travel from one prism to another cyclically.

Note that phase portraits of even-dimensional analogues of system (1) have quite different structures.

These geometric constructions allow us to obtain the following analytical results:

3. Sufficient conditions of stability of some of these cycles in Q.

4. Sufficient conditions of non-uniqueness of cycles in the cases when $2k+1$ is not a prime number, and the dynamical system (1) is symmetric with respect to cyclic permutations of the variables, i.e., when all the functions f_i coincide: $f_i = f$.

Here, for a step function f, we construct invariant two-dimensional sub-manifolds of the phase portrait of this system and show non-uniqueness of the cycles for $k = 2$, $k = 3$. In both cases, only one of these cycles is contained in Q. Bifurcations of these cycles were studied as well.

Multitemporal wave equations and mean value operators

Fulton B. Gonzalez
Tufts University, USA
e-mail: fulton.gonzalez@tufts.edu

Let $X = G/K$ be a (compact, noncompact, or flat) Riemannian symmetric space. Let $\Gamma : \mathbb{D}(X) \to \mathbb{D}_W(\mathfrak{a})$ be the Harish-Chandra isomorphism. (If X is flat, we replace Γ by a simple restriction map.) Let w denote the order of the Weyl group W. For $f_1, \ldots, f_w \in C^\infty(X)$, consider the multitemporal system

$$D_x\, u(x,H) = \Gamma(D)_H\, u(x,H) \qquad\qquad (D \in \mathbb{D}(X))$$

for $u \in C^\infty(X \times \mathfrak{a})$, with initial data

$$\partial(p_j)_H\, h(x,0) = f_j(x) \qquad\qquad (j = 1, \ldots, w).$$

This system was introduced by Semenov-Tjan-Shansky and developed by Lax, Phillips, Shahshahani, and others. We review solution methods and properties of solutions, including Huygens' principle, relation to the horocycle Radon transform, and hypergeometric shift operators.

Global normal forms for Shubin type pseudodifferential operators

Todor Gramchev
Università di Cagliari, Italy
e-mail: todor@unica.it

We study the reduction to normal forms of classes of second order pseudodifferential operators of Shubin type by global transformations acting continuously in Gelfand-Shilov spaces.

Integral transforms defined by intrinsic geometry of Riemannian manifolds

Eric Grinberg
University of Massachusetts Boston, USA
e-mail: eric.grinberg@umb.edu

We consider integral transforms on a Riemannian manifold M which are defined using the intrinsic geometry of M. The best known such operator is the x-ray transform, which integrates over geodesics. In some cases, especially when there is symmetry present, it is possible to define integral transforms over surfaces that are thickenings of geodesics. In some such cases, one can attain injectivity results for the underlying operators, and in still more special cases, e.g., symmetric spaces, inversion formulas may be obtained.

Boundary behaviour of one-parameter semigroups and evolution families

Pavel Gumenyuk
Università di Roma "Tor Vergata", Italy
e-mail: gumenyuk@mat.uniroma2.it

A one-parameter semigroup of holomorphic functions in the unit disk is a continuous homomorphism from the additive semigroup of non-negative reals to the set of all holomorphic self-maps of the unit disk, endowed with the composition operation and the topology of locally uniform convergence. The theory of one-parameter semigroups is a classical topic of Complex Analysis: they appear in the iteration theory in the unit disk as fractional iterates, in Operator Theory they have been studied in connection with composition operators, in Probability Theory one-parameter semigroups turn out to be related to the embedding problem for branching processes.

The first part of the talk is devoted to the local boundary behaviour of one-parameter semigroups. We present some classical and new results concerning boundary fixed points of one-parameter semigroups, their angular and unrestricted limits on the unit circle. We extend two remarkable statements due to M. D. Contreras, S. Díaz-Madrigal and Ch. Pommerenke. First of them asserts that in contrast to an arbitrary univalent holomorphic self-map of the unit disk, elements of one-parameter semigroups have angular limits at every point on the unit circle. The other statement says that at every(super) repulsive boundary fixed point there exists, in fact, the unrestricted limit, so that the semigroup, being extended to the unit circle by angular limits, becomes continuous at such points. We prove that this is also the case for the boundary Denjoy-Wolff point.

The second part of the talk deals with a non-autonomous analogue of one-parameter semigroups, the so-called evolution families in the unit disk. This notion is of crucial importance for the much celebrated Loewner Theory. We present a new result on the correspondence between common regular boundary fixed points of an evolution family and the boundary singularities of the right-hand side in the generalized Loewner equation generating this evolution family. This is a joint result with Professors Filippo Bracci, Manuel D. Contreras and Santiago Díaz-Madrigal.

Persistency of analyticity for quasi-linear wave equations: an energy-like approach

Yanqiu Guo
Weizmann Institute, Israel
e-mail: yanqiu.guo@weizmann.ac.il

I will briefly introduce the theory of Gevrey classes and employ it to study the persistence of analyticity of solutions to quasi-linear wave equations with real analytic nonlinearity. Specifically, we prove that the solution remains spatial analytic with respect to some of its spatial variables during its whole life-span, provided the initial data is analytic with respect to these spatial variables. In addition, we find a lower bound for the radius of the spatial analyticity of the solution that might shrink either algebraically or exponentially, in time, depending on the structure of the nonlinearity. This is a joint work with Edriss S. Titi.

Lagrange polynomials, reproducing kernels and Markov's polynomial inequality

Lawrence A. Harris
University of Kentucky Lexington, USA
e-mail: larry@ms.uky.edu

In this talk, we consider sharp upper bounds on the norm of the n-th order Gateaux and Frechet derivatives for polynomials with a given bound on the closed unit ball of real normed linear spaces. We show that V. A. Markov's classical bounds continue to hold for the Gateaux derivatives and that better estimates hold that depend on the value of polynomial at the point where the derivative is taken and at the origin.

These results are established by a new technique that uses an explicit formula for bivariate Lagrange polynomials for certain nodes in the plane and a corresponding Christoffel-Darboux formula. Bounds on the derivatives are obtained directly from bounds on the derivatives of the Lagrange polynomials.

On fractional Poincaré inequalities

Ritva Hurri-Syrjanen
Helsingin yliopisto, Finland
e-mail: ritva.hurri-syrjanen@helsinki.fi

In this talk, we discuss recent results on fractional Poincaré inequalities. We show that fractional Poincaré inequalities and fractional Sobolev–Poincaré inequalities hold in bounded John domains and, in particular, in bounded Lipschitz domains. Joint work with Antti V. Vähäkangas.

Intersections of sets, group actions and Erdos–Falconer problems

Alex Iosevich
University of Rochester, USA
e-mail: iosevich@math.rochester.edu

We shall consider Erdos–Falconer type problems in geometric measure theory where invariance with respect to a given group of transformations play an important role. We shall derive a general Mattila integral with respect to these group actions and use them to obtain sharp exponents for the corresponding geometric problems.

Explicit reconstruction of homogeneous isolated hypersurface singularities from their Milnor algebras

Alexander Isaev
Australian National University, Australia
e-mail: Alexander.Isaev@anu.edu.au

By the Mather-Yau theorem, a complex hypersurface germ V with isolated singularity is completely determined by its moduli algebra $A(V)$. The proof of the theorem does not provide an explicit procedure for recovering V from $A(V)$, and finding such a procedure is a long-standing open problem. In this talk, I will present an explicit way for reconstructing V from $A(V)$ up to biholomorphic equivalence under the assumption that the singularity of V is homogeneous, in which case $A(V)$ coincides with the Milnor algebra of V.

Non-exploding analytic diffusions in the unit disk

Georgy Ivanov
Universitetet i Bergen, Norway
e-mail: Georgy.Ivanov@math.uib.no

We apply general Loewner theory (Bracci et al.) to characterize analytic Ito diffusions in the unit disk for which solutions always stay in the given domain; we also prove the dual result using decreasing Loewner chains. We characterize infinitesimal generators giving rise to slit semigroups and, consequently, describe a general form of diffusions generating slit evolutions.

Recent results about uniqueness and continuous dependence in the Cauchy problem for backward-parabolic operators with low-regular coefficients

Christian Jäh
Technische Universität Bergakademie Freiberg, Germany
e-mail: christian.jaeh@math.tu-freiberg.de

We consider the Cauchy problem for backward-parabolic operators

$$\mathcal{P}u = \partial_t u + \sum_{i,j=1}^{n} \partial_{x_i}(a_{i,j}(t,x)\partial_{x_j}u) + \sum_{k=1}^{n} b_k(t,x)\partial_k u + c(t,x)u$$

and look for sufficient and necessary conditions to ensure the uniqueness of the solutions or the continuous dependence [5] of the solutions on the Cauchy data. We are especially interested in the connections between the regularity of the principal part coefficients and the mentioned properties. It is almost classical that the questions about uniqueness and stability have a positive answer if $a_{i,j}(t,x)$ are Lipschitz continuous with respect to time and bounded with respect to the spatial variable [1]. We follow two possibilities to weaken the Lipschitz condition:

Local irregularity: Suppose that $a_{ij} = a_{ij}(t)$ with $|F(t)d_t a_{ij}(t)| \leq C_{small}$, where F is a suitable function like $F(t) = t$ or $F(t) = t^2$.

Global irregularity: Suppose that $a_{ij} = a_{ij}(t,x) \in C^{\mu}([0,T], L^{\infty}(\mathbb{R}^n) \cap L^{\infty}([0,T], C^{\omega}(\mathbb{R}^n))$, where we investigate also the possible interactions between ω and μ. We will illustrate the necessity of our conditions by suitable counterexamples. To prove our results we will use the Carleman estimate method and to prove suitable Carleman estimates we will use, e.g, Bony's para-differential calculus. The results about local irregularity are connected to singular Carleman weight functions.

References

[1] J.-L. Lions and B. Malgrange, Sur l'unicit rctrograde dans les problmes mixtes paraboliques, *Mathematica Scandinavica*, 1960, 8, pp. 277-286.

[2] D. Del Santo and Ch. P. Jäh, Non-uniqueness and uniqueness in the Cauchy problem of elliptic and backward-parabolic equations, 26pp. In: *Progress in Partial Differential Equations - Asymptotic Profiles, Regularity and Well-Posedness*, Springer, ISBN: 978-3-319-00124-1.

[3] D. Del Santo, C. P. Jäh and M. Paicu, Backward uniqueness for parabolic operators with non-Lipschitz coefficients, **in preparation**.

[4] D. Del Santo, C. P. Jäh and M. Paicu, Backward uniqueness for parabolic operators with non-Lipschitz coefficients, **in preparation**.

[5] D. Del Santo and M. Prizzi, Continuous dependence for backward parabolic operators with Log-Lipschitz coefficients, *Mathematische Annalen*, 2009, 345(1), pp. 213-243.

Generalized typically-real functions

Stanislawa Kanas
Politechnika Rzeszowska, Poland
e-mail: skanas@prz.edu.pl

Let $f(z) = z + a_2 z^2 + \cdots$ be regular in the unit disk and real valued if and only if z is real and $-1 < z < 1$. Then $f(z)$ is said to be typically-real function. Rogosinski has shown the necessary and sufficient condition for a regular function to be typically-real. The main purpose of the paper is to consider the generalized typically-real functions defined via the generating function of the generalized Meixner-Pollaczek polynomials

$$G^{\lambda}(x;\theta,\psi;z) = \frac{1}{(1-ze^{i\theta})^{\lambda-ix}(1-ze^{i\psi})^{\lambda+ix}} = \sum_{n=0}^{\infty} P_n^{\lambda}(x;\theta,\psi)z^n, \ |z| < 1.$$

Stieltjes function and Hurwitz stable entire functions

Victor Katsnelson
Weizmann Institute, Israel
e-mail: victor.katsnelson@weizmann.ac.il

The concept of stability, originally introduced for polynomials, will be extended to apply to the class of entire functions. This generalization will be called Hurwitz stablility and the class of Hurwitz stable functions will serve as the main focus of this paper. A first theorem will show how, given a function of either of the Stieltjes classes, a Hurwitz stable function might be constructed. A second approach to constructing Hurwitz stable functions, based on using additional functions from the Laguerre–Pólya class, will be presented in a second theorem.

On solvability of multi-order parabolic systems

Oleg Kelis
University of Haifa, ORT Braude College, Karmiel, Israel
e-mail: olegkelis@braude.ac.il

In the present work, we investigate the following system of differential equations:

$$\mathcal{A}(D,\partial_t)=\partial_t\mathbf{u}+A\mathbf{u}=\mathbf{f}\quad\text{in}\quad Q:=\Omega\times[0,T],\quad T<\infty, \tag{1}$$

where A is a multi-order operator elliptic in the sense of Douglis–Nirenberg with different diagonal orders [1].

A parameter-ellipticity condition for these type of operators was invented by Kozhevnikov [1] for operators over a compact boundaryless manifold and later [2] in the case of boundary value problems.

Using Kozhevnikov's method [2] and some results by Lions-Magenes [3], we prove solvability of the parabolic multi-order initial-boundary value problems in appropriate anisotropic Sobolev-type spaces.

Advisors: Prof. Alexander Kozhevnikov, Prof. Vladimir Rovenski.

References

[1] Kozhevnikov, A., Spectral problems for pseudo-differential and systems elliptic in the Douglis-Nirenberg sense, and their applications, *Mat. Sb.* **92** (134) (1973), p. 68–88; English transl. in Math. USSR Sb. **21** (1973), p. 63–90.

[2] Kozhevnikov, A., Parameter-ellipticity for mixed-order systems elliptic in the sense of Petrovskii, *Commun. Appl. Anal.* **5** (2001), p. 277–291.

[3] Lions, J. L., Magenes, E., Non-Homogeneous Boundary Value Problems and Applications, vol. 2, Springer, Berlin 1972.

An overdetermined boundary value problem

Dmitry Khavinson
University of South Florida, USA
e-mail: dkhavins@usf.edu

We shall discuss a recent solution of a problem to characterize all unbounded domains in the plane that admit positive harmonic functions vanishing on the boundary with a normal derivative equal to one there. The solution for simply connected domains is obtained in a joint work with E. Lundberg and R. Teodorescu. Even more recently, M. Traizet announced a complete solution for domains of higher connectivity that makes an unexpected connection with the deep results in the theory of minimal surfaces. The higher dimensional version of the problem however mostly remains open.

Extremal properties associated with the generalized Loewner differential equation in $\mathbb{C}^n$

Gabriela Kohr
Universitatea Babeş-Bolyai, Romania
e-mail: gkohr@math.ubbcluj.ro

In this talk, we survey recent results related to extreme points, support points and reachable families of holomorphic mappings generated by the generalized Loewner differential equation on the unit ball B^n in $\mathbb{C}^n$. Certain applications and conjectures are also considered.

For a linear operator $A \in L(\mathbb{C}^n)$, let $k_+(A)$ be the upper exponential index of A and let $m(A) = \min\{\mathrm{Re}\langle A(z), z\rangle : \|z\| = 1\}$. Under the assumption $k_+(A) < 2m(A)$, we consider the family $S_A^0(B^n)$ of mappings which have A-parametric representation, i.e. $f \in S_A^0(B^n)$ iff there exists an A-normalized univalent subordination chain $f(z,t)$ such that $f = f(\cdot, 0)$ and $\{e^{-tA} f(\cdot, t)\}_{t\geq 0}$ is a normal family on B^n. We are concerned with extreme points and support points associated with the compact family $S_A^0(B^n)$. These results generalize to higher dimensions related results due to Pell and Kirwan. We also present an n-dimensional version of an extremal principle due to Kirwan and Schober, and give applications related to distortion and coefficient bounds for $S_{I_n}^0(B^n)$.

In the second part of the talk, we use ideas from control theory to consider extremal problems related to bounded mappings in $S_A^0(B^n)$. For this aim, we investigate the (normalized) time-$\log M$-reachable family $\tilde{\mathcal{R}}_{\log M}(\mathrm{id}_{B^n}, \mathcal{N}_A)$ generated by the Carathéodory mappings, where $M \geq 1$ and $k_+(A) < 2m(A)$. Every mapping f in this reachable family can be imbedded as the first element of an A-normalized univalent subordination chain $f(z,t)$ such that $\{e^{-tA} f(\cdot, t)\}_{t\geq 0}$ is a normal family and $f(\cdot, \log M) = e^{A \log M} \mathrm{id}_{B^n}$. We present a density result related to the family $\tilde{\mathcal{R}}_{\log M}(\mathrm{id}_{B^n}, \mathcal{N}_A)$, which involves the subset $\mathrm{ex}\mathcal{N}_A$ of $\mathcal{N}_A$ consisting of extreme points. These results are generalizations to $\mathbb{C}^n$ of well known results due to Loewner, Pommerenke and Roth. We are also concerned with extreme points and support points associated with compact families generated by extension operators.

This talk is based on joint work with Ian Graham (Toronto), Hidetaka Hamada (Fukuoka) and Mirela Kohr (Cluj-Napoca).

Poisson problems for semilinear Brinkman systems in Lipschitz domains

Mirela Kohr
Universitatea Babeş-Bolyai, Romania
e-mail: mkohr@math.ubbcluj.ro

The purpose of this talk is to combine a layer potential analysis with a fixed point theorem in order to show the existence of solutions for Poisson problems associated to semilinear Brinkman systems on a bounded Lipschitz domain $\mathfrak{D} \subseteq \mathbb{R}^n$ $(n \geq 2)$ with Dirichlet or Robin boundary conditions and the given data in Sobolev and Besov spaces. The Dirichlet problem for the Darcy–Forchheimer–Brinkman system is also discussed.

Stability and separation in volume comparison problems

Alexander Koldobsky
University of Missouri-Columbia, USA
e-mail: koldobskiya@missouri.edu

We prove stability in several volume comparison problems and show how these results imply hyperplane inequalities for different classes of convex bodies.

Stochastic dynamics of a continuum particle system with dispersal and competition: micro- and meso-scopic description

Jurij Kozicki
Uniwersytet Marii Curie-Skłodowskiej w Lublinie, Poland
e-mail: jkozi@hektor.umcs.lublin.pl

A Markov dynamics of a system of point particles in Rd is described at micro- and mesoscopic levels. The particles reproduce themselves at distant points (dispersal) and die, independently and under the influence of each other (competition). The microscopic description is based on an infinite chain of equations for correlation functions, similar to the BBGKY hierarchy used in the Hamiltonian dynamics of continuum particle systems. The meso-scopic description is based on a Vlasov-type kinetic equation for the particles density obtained from the mentioned chain via a scaling procedure. The main conclusion of the microscopic theory is that the competition can prevent the system from clustering, which makes its description in terms of densities reasonable. A possible homogenization of the solutions to the kinetic equation in the long-time limit is also discussed.

Sharp real-part theorems for derivatives of analytic functions

Gershon Kresin
Ariel University of Samaria, Israel
e-mail: kresin@ariel.ac.il

It is assumed that the boundary values of the real part of analytic functions in the unit disk $\mathbb{D}$ and the upper half-plane $\mathbb{C}_+$ are in L^p.

Representations for the sharp coefficient in an estimate of the modulus of the n-th derivative of analytic functions in $\mathbb{D}$ and $\mathbb{C}_+$ are obtained. The maximum of a bounded factor in the representation of the sharp coefficient for analytic functions in $\mathbb{D}$ is found. Thereby, a pointwise estimate of the modulus of the n-th derivative of an analytic function in $\mathbb{D}$ with a best constant is given. The representation for the sharp coefficient in the estimate of the modulus of the n-th derivative of analytic functions in $\mathbb{C}_+$ is concretized for some n and p. In particular, for $p = \infty$ and for derivatives of odd order of analytic functions in $\mathbb{C}_+$, an explicit formula for the sharp coefficient is found.

A limit relation for the sharp coefficient in a pointwise estimate for the modulus of the n-th derivative of an analytic function in a disk is found as the point approaches the boundary circle. The relation in question contains the sharp constant from the estimate of the modulus of the n-th derivative of an analytic function in $\mathbb{C}_+$. As a corollary, a limit relation for the modulus of the n-th derivative of an analytic function with the bounded real part is obtained in a domain with smooth boundary.

Strengthened Grunsky and Milin inequalities

Samuel Krushkal
Bar-Ilan University, Israel
e-mail: krushkal@math.biu.ac.il

The method of Grunsky inequalities has many applications and was extended in many directions, even to bordered Riemann surfaces. However, unlike the case of functions univalent in the disk, a quasiconformal variant of this theory has not been developed so far.

In the talk, I present essential improvements of the basic facts concerning the classical Grunsky inequalities for univalent functions on the disk and extend these results to arbitrary quasiconformal disks. Various applications will be given.

The nodal count mystery

Peter Kuchment
Texas A&M University, USA
e-mail: pkuchment@gmail.com

In this talk we address the nodal count (i.e., the number of nodal domains) for eigenfunctions of Schroedinger operators with Dirichlet boundary conditions in bounded domains (billiards). The classical Sturm theorem claims that in dimension one, the nodal and eigenfunction counts coincide: the n-th eigenfunction has exactly n nodal domains. The Courant Nodal Theorem claims that in any dimension, the number of nodal domains of the n-th

eigenfunction cannot exceed n. However, it follows from a stronger upper bound by Pleijel that in dimensions higher than 1 the equality can hold for only finitely many eigenfunctions. Thus, in most cases a "nodal deficiency" arises. Moreover, examples are known of eigenfunctions with an arbitrarily large index n that have just two nodal domains. One can say that the nature of the nodal deficiency had not been understood. We show that, under some genericity conditions, the answer can be given in terms of a functional on an infinite dimensional variety of partitions of the billiard, whose critical points correspond exactly to the nodal partitions and Morse indices coincide with the nodal deficiencies. This is joint work with G. Berkolaiko (Texas A&M University) and U. Smilansky (Weizmann Institute) in GAFA 2012.

The common fixed point set of commuting holomorphic mappings in Cartesian products of Banach spaces

Tadeusz Kuczumow
Uniwersytet Marii Curie-Skłodowskiej w Lublinie, Poland
e-mail: tadek@hektor.umcs.lublin.pl

We establish a theorem regarding the common fixed point of commuting holomorphic self-mappings of domains in Cartesian products of Banach spaces. The proof is based on a method due to R. E. Bruck.

Photo- and thermo- acoustic tomography in the presence of reflecting boundaries

Leonid Kunyansky
University of Arizona, Tucson, USA
e-mail: leonk@math.arizona.edu

Most of the known theoretical and algorithmic results pertaining to the inverse problem of the thermo- and photo- acoustic tomography (TAT/PAT) are based on the assumption that the acoustic waves propagate in free space. However, neglecting the reflections of the waves from the detectors is not always possible. For example, when the detectors are formed by optically scanned solid plates surrounding the object, the multiple reflections of the acoustic waves from the plates need to be taken into account. In this case, the forward problem is accurately modeled by the wave equation with the Neumann boundary conditions on the walls of the resonant cavity formed by the detecting surfaces. The energy of waves in such a cavity does not decay in time (if we neglect the absorption) which renders inapplicable classical results for TAT/PAT that rely on the fast decay of waves within the region surrounded by the detectors. In the talk, we will discuss theoretical issues arising in the problem of TAT/PAT reconstruction in a reverberant cavity and will present reconstruction algorithms applicable for a rectangular and spherical cavities. (This is joint work with B.T. Cox and B. Holman)

Multi-tiling and Riesz bases

Nir Lev
Bar-Ilan University, Israel
e-mail: levnir@math.biu.ac.il

Let S be a bounded, Riemann measurable set in $\mathbb{R}^d$, and L be a lattice. By a theorem of Fuglede, if S tiles $\mathbb{R}^d$ with translation set L, then S has an orthogonal basis of exponentials. We show that, under the more general condition that S multi-tiles $\mathbb{R}^d$ with translation set L, S has a Riesz basis of exponentials. The proof is based on Meyer's quasicrystals. Joint work with Sigrid Grepstad.

Domains with conically accessible boundary in multi-dimensional case

Piotr Liczberski
Politechnika Łódzka, Poland
e-mail: piliczb@p.lodz.pl

The lecture will be based on joint a paper [6] with V. V. Starkov. It says usually that a domain $\Omega \subset \mathbb{R}^n$ satisfies the "cone condition" if for every $p \in \Omega$ it includes a closed circular cone $K(p, e(p), \alpha, r)$ with the vertex at the point $p \in \mathbb{R}^n$, an axis vector $e(p)$ and some fixed opening $\alpha\pi$, $\alpha \in (0, 1)$ and height $r \in (0, \infty]$.

The cone property and its generalizations are the main tool for solving very important various mathematical problems, for instance some problems in the theory of integral representations of functions, some problems concerning the imbedding theorems, and some problems occuring in the investigation of boundary behaviour of functions (see e.g. [1], [2, Chapt. 1, Par. 8], [4], [9]).

During the lecture, we will consider domains $\Omega \subset \mathbb{R}^n$ with a property similar to the above cone property. We say that a domain $\Omega \subset \mathbb{R}^n$, including the origin, is α-accessible, $\alpha \in [0, 1)$, if for every point $p \in \partial\Omega$ there exists a number $r = r(p) > 0$ such that the cone $K(p, p, \alpha, r) > 0$ is included in $\mathbb{R}^n \setminus \Omega$. We will give a few geometric properties of α-accessible domains, in particular the following: if $\Omega \subset \mathbb{R}^n$ is an α-accessible domain, $\alpha \in (0, 1)$, then for every $p \in \partial\Omega$ and every $\eta \in (0, \alpha)$ there exists a number $r = r(p, \eta) > 0$, such that the bounded cone $K(p, -p, \eta, r)$ is included in Ω. We will give also an analytic test for α-accessibility of domains with smooth boundaries. As an application of the above, we demonstrate a solution of the following problem, originated in [5]: characterize all α-accessible domains in $\mathbb{C}^N$ which are biholomorphic to the Euclidean ball. Such considerations in $\mathbb{C}^n$ are continuation of some investigations in the complex plane $\mathbb{C}$ (see [8], [3], [7]).

References

[1] Adams R. A., Fournier J., Cone conditions and properties of Sobolev spaces, *J. Math. Anal. Appl.* **61** (1977), 713–734.

[2] Besov O. V., Ilin V. P., Nikolskij S. M., Integral Representations of Functions and Imbedding Theorems, vol I, II, John Wiley & Sons, New York-Toronto, Ontario-London, 1978,1979.

[3] Brannan D. A., Kirwan W. E., On some classes of bounded univalent functions, *J. London Math. Soc.* **2** (1969), 431–443.

[4] Dolzhenko E. P., Boundary properties of arbitrary functions, *Izv. Acad. Sci. SSSR, Ser. Math.* **31** (1967), 3–14.

[5] Kohr G., Liczberski P., On strongly starlikeness of order alpha in several complex variables, *Glasnik Math.* **33 (53)** (1998), 185–198.

[6] Liczberski P., Starkov V. V., Domains in $\mathbb{R}^n$ with conical accessible boundary, submitted.

[7] Ma W., Minda D., An internal geometric characterization of strongly starlike functions, *Ann. Univ. Mariae Curie-Skłodowska*, Sect. A, **45** (1991), 89-97.

[8] Stankiewicz J., Quelques problèmes extrémaux dans les classes α-angulairement étoilées, *Ann. Univ. Mariae Curie-Skłodowska* **20** (1966), 59-75.

[9] Zaremba S., Sur le principle de Dirichlet, *Acta Math.* **34** (1911), 293-316.

Fourier transform versus Hilbert transform

Elijah Liflyand
Bar-Ilan University, Israel
e-mail: liflyand@gmail.com

We discuss several problems where the interplay of the two transforms is either an interesting and important ingredient of the proofs or appears in assumptions or assertions. The obtained results are concerned with the integrability of the Fourier transform. Among the considered spaces stand out and most actively used the Hardy space and that of functions of bounded variation.

Extremal problems for polynomials generate extremal problems for Paley-Wiener space

Doron Lubinsky
Georgia Institute of Technology, USA
e-mail: lubinsky@math.gatech.edu

When suitably scaled, limits of polynomials become entire functions of exponential type in a Paley-Wiener space. This observation has been used in approximation theory for perhaps a century. We present a new result in this theme, showing how asymptotics of L_p Christoffel functions for polynomials lead to an extremal problem in a Paley-Wiener space. In some sense, this problem was considered by Boas and Korevaar decades ago. This is joint work with Eli Levin of the Open University of Israel.

Nonlinear wave equations with variable coefficients

Sandra Lucente
Università degli Studi di Bari, Italy
e-mail: lucente@dm.uniba.it

In the present talk, we discuss the global existence theory for some wave equation of kind

$$u_{tt}(t,x) - a(t)\Delta u(t,x) + b(t)u_t(t,x) = \Gamma(t,x)f(u(t,x)), \quad t > 0, \quad x \in \mathbb{R}^n,$$

with positive a, b and $|f(u)| \simeq |u|^p$ with $p > 1$.

It is well known that such kind of results depends on the size of the initial data and on the growth of the nonlinear term. In some lucky situations, there exist a range for p in which a global existence result is established and a complementary range of p in which one gains a non-existence result. These ranges are intervals and the separator $\overline{p}$ between such ranges is called *critical exponent*. The same equation can admit different critical exponents according to the kind of solution we are looking for. For constant $a = b = \Gamma$, the critical exponents depend on the space dimension n. In the variable coefficient setting, the critical exponents may depend on the growth or the zero order of the coefficients.

After a brief review of the literature on this problem, we shall present in detail

- the existence of the global small data solution for constant a, γ and *effetive* $b(t)$, see [1];
- the correspondent non-existence result for weak solutions, see [2];
- some partial results for variable $a(t)$.

References

[1] M. D'Abbicco, S. Lucente, M. Reissig, *Semilinear wave equations with effective damping*, To appear on Chin. Ann. of Math. Ser. B ArXiv: 1210.3493v1.

[2] M. D'Abbicco, S. Lucente, A modified test function method for damped wave equations, Submitted ArXiv:1211.0453, (2012).

Logarithmic convexity for discrete harmonic functions

Eugenia Malinnikova
Norges teknisk-naturvitenskapelige universitet, Norway
e-mail: eugenia@math.ntnu.no

Logarithmic convexity of square norms of continuous harmonic functions over a family of concentric balls or parallel hyperplanes are well known and have important applications. For example, the three balls theorem is an immediate consequence of such logarithmic convexity.

In the talk we consider the discrete Laplace operator on an n-dimensional standard lattice. It is similar to the continuous Laplace operator in many respects but lacks for example the rotational symmetry. We discuss some known and new logarithmic convexity estimates for discrete harmonic functions. The talk is based on joint works with M. Guadie.

Bounds for eigenfunctions of the Laplacian on noncompact Riemannian manifolds

Vladimir Maz'ya
Linköpings universitet, Sweden
e-mail: vladimir.mazya@liu.se

We deal with eigenvalue problems for the Laplacian on noncompact Riemannian manifolds M of finite volume. Sharp conditions ensuring $L^q(M)$ and $L^\infty(M)$ bounds for eigenfunctions are exhibited in terms of either the isoperimetric function or the isocapacitary function of M.

Jordan Theory and Holomorphic Dynamics

Pauline Mellon
University College Dublin, Ireland
e-mail: pauline.mellon@ucd.ie

Studies of the dynamics of a holomorphic self-map on a bounded convex domain often rely on the domain having additional convexity properties. For bounded symmetric domains this approach is not possible since, apart from the Hilbert ball, these domains lack the necessary extra convexity. Their algebraic structure, however, makes strong Jordan techniques available and earlier work in this direction has produced Denjoy-Wolff type results there. We show here how Jordan theory can be used to study the iterates, (g^n), of a fixed-point free biholomorphic self-map, g, on a bounded symmetric domain, B. In infinite dimensions, (g^n) does not necessarily converge, even for the Hilbert ball, as was shown by Stachura. On the other hand, each g can be written $g = g_a \circ T$, for a linear isometry T and a transvection g_a $(a = g(0))$ and we can determine the dynamics of g_a. Namely, (g_a^n) converges locally uniformly on B if, and only if, a is regular, in which case, the limit is a holomorphic map of B onto a boundary component (surprisingly though, generally not the boundary component of $\frac{a}{\|a\|}$). These results are new even in finite dimensions where every element is regular.

Sharp isoperimetric inequalities and model spaces for Curvature-Dimension-Diameter condition

Emanuel Milman
The Technion - Israel Institute of Technology, Israel
e-mail: emanuel.milman@gmail.com

In recent years, thanks to the works of Sturm, Lott–Villani and others, the relevance of optimal-transport to the characterization of metric-measure spaces admitting a generalized Ricci curvature lower-bound and generalized dimension upper-bound, has become apparent. Various properties of a space satisfying such a Curvature-Dimension condition may be studied. In this work, we obtain new sharp isoperimetric inequalities in the more traditional setting of a Riemannian manifold equipped with a probability measure, whose generalized Ricci curvature is bounded from below (possibly negatively), and generalized dimension and diameter of the convex support are bounded from above (possibly infinitely).

Our inequalities are *sharp* for sets of any given measure and with respect to all parameters (curvature, dimension and diameter). Moreover, for each choice of parameters, we identify the *model spaces* which are extremal for the isoperimetric problem. In particular, we recover the Gromov–Lévy and Bakry–Ledoux isoperimetric inequalities, which state that whenever the curvature is strictly *positively* bounded from below, these model spaces are the n-sphere and Gauss space, corresponding to generalized dimension being n and ∞, respectively. In all other cases, which seem new even for the classical Riemannian-volume measure, it turns out that there is no *single* model space to compare to, and that a simultaneous comparison to a natural *one parameter family* of model spaces is required, nevertheless yielding a sharp result. Time permitting, we will mention some resulting Sobolev and log-Sobolev inequalities on such spaces.

Harmonic analytic and geometric measure theoretic methods in several complex variables

Irina Mitrea
The Temple University, Philadelphia, USA
e-mail: imitrea@temple.edu

Practice has shown that the combination of Harmonic Analysis, Geometric Measure Theory and Complex Analysis is an extremely fertile and potent mix in the complex plane, with many notable achievements whose degree of technical sophistication is breathtaking. In sharp contrast with these successes, the case of several complex variables has been very little explored from the perspective of the latest advances of Harmonic Analysis and Geometric Measure Theory. The aim of this talk is to discuss some recent progress in this direction and illustrate how tools and methods from Harmonic Analysis and Geometric Measure Theory may yield a qualitative upgrade of some of the most fundamental results in several complex variables.

Lightcone estimates for spacetime curvature in general relativity

Vincent Moncrief
Yale University New Haven, Connecticut, USA
e-mail: vincent.moncrief@yale.edu

For various nonlinear fields propagating in curved, four-dimensional spacetimes one can write an exact integral formula for the field at an arbitrary point in terms of an integral over the past light cone from that point to an initial, Cauchy hypersurface and an additional integral over the intersection of the cone with this surface. This is possible even when Huygens' principle fails to hold since the integral expression over the past cone involves the unknown itself and thus is not an actual 'representation formula' for the solution. It can nevertheless provide the basis for deriving certain bounds on the nonlinear evolution and can often be used to prove no-blow-up results if certain associated 'energies' and their corresponding fluxes are under control. I shall describe these 'lightcone estimates' for nonlinear scalar waves, Yang-Mills fields and for the curvature of a solution to Einstein's field equations.

The Hilbert transform, Perron–Frebonius operators and the Klein–Gordon equation

Alfonso Montes-Rodríguez
Universidad de Sevilla, Spain
e-mail: amontes@us.es

The unidimensional (normalized) Klein–Gordon equation is

$$\partial_x \partial_y f = f.$$

Let α, β be positive numbers. Consider the lattice cross

$$\Lambda_{\alpha,\beta} = \{(\alpha n, 0) : n \in \mathbb{Z}\} \cup \{(0, \beta\mathbb{Z}) : n \in \mathbb{Z}\}.$$

Hedenmalm and Montes-Rodríguez have shown that if the Fourier transform of f is an absolutely continuous measure and f vanishes on $\Lambda_{\alpha,\beta}$, then f must be the zero function if and only if $\alpha\beta \leq 1$. This problem is equivalent to show that

$$e^{\alpha n 2\pi i z},\ n \in \mathbb{Z} \quad \text{and} \quad e^{\beta m 2\pi i/z},\ m \in \mathbb{Z}$$

span a weakly dense subspace in $L^\infty(\mathbb{R})$. However, it remained open the problem whether

$$e^{\alpha n 2\pi i z},\ n = 0, 1, 2, ... \quad \text{and} \quad e^{\beta m 2\pi i/z},\ m = 0, -1, -2, ... \tag{2}$$

span a weakly dense subspace of $H^\infty(\mathbb{C}^+)$. The latter problem is equivalent to the fact that if a solution of the Klein–Gordon equation whose Fourier transform is an absolutely continuous measure and vanishes on $(\alpha n, 0)$ and $(0, -n\beta)$, $n = 0, 1, 2, ...$, then f also vanishes on the fourth quadrant. Again, we have recently been able to show that the functions in (2) span a weakly dense subspace of $H^\infty(\mathbb{C}^+)$ if and only if $\alpha\beta \leq 1$. The proof reduces to a delicate analysis of how the Hilbert transform intertwines with certain Perron–Frobenius operator. There are several consequences. For instance, the algebra $\mathcal{A}$ spanned by $\exp(\pi(1+z)/(z-1))$ and $\exp(\pi(z-1)/(1+z))$ is weakly dense in H^∞ of the unit disk. As a consequence, the lattice of $\mathcal{A}$-invariant subspaces in the Hardy spaces H^p, $1 \leq p < \infty$, coincides with the shift invariant subspaces. We will also discuss other related issues. Joint work with Håkan Hedenmalm.

Evolving hypersurfaces by their inverse null mean curvature

Kristen Moore
Universität Potsdam, Germany
e-mail: kristen.moore@aei.mpg.de

We introduce a new second order parabolic evolution equation where the speed is given by the reciprocal of the null mean curvature. This flow is a generalisation of inverse mean curvature flow and it is motivated by the study of black holes and mass/energy inequalities in general relativity. We present a theory of weak solutions using level-set methods and an appropriate variational principle, and outline a natural application of the flow as a variational approach to constructing marginally outer trapped surfaces (MOTS), which play the role of quasi-local black hole boundaries in general relativity.

Holomorphic extension of functions along the finite families of complex lines in a ball of $\mathbb{C}^n$

Simona Myslivets
Siberian Federal University, Krasnoyarsk, Russia
e-mail: sMyslivets@sfu-kras.ru

In this talk we consider continuous functions given on the boundary of a ball B in $\mathbb{C}^n$, $n > 1$ and having a one-dimensional property of holomorphic extension along the families of complex lines, passing through a finite number of points of B. We study the problem of existence of holomorphic extension of such functions in the ball B.

Spherical mean transform from a PDE point of view

Linh Nguyen
University of Idaho, USA
e-mail: lnguyen@uidaho.edu

We consider the spherical mean transform on $\mathbb{R}^n$. The transform is characterized by the Euler-Poisson-Darboux equation. By looking at the spherical harmonic expansions, we obtain a system of $1 + 1$-dimension hyperbolic equations. Using these equations, we discuss two known problems. The first one is a local uniqueness problem investigated by M. Agranovsky and P. Kuchment, [*Memoirs on Differential Equations and Mathematical Physics*, **52** (2011), 1–16]. We present a proof which only involves simple energy arguments. The second problem is to characterize the kernel of spherical mean transform on annular regions, which was studied by C. Epstein and B. Kleiner [*Comm. Pure Appl. Math.*, **46(3)** (1993), 441–451]. We present a short proof that simultaneously provides the necessity and sufficiency for the characterization. As a consequence, we derive a reconstruction procedure for the transform with additional interior (or exterior) information.

We also discuss how the approach works for the hyperbolic and spherical spaces.

High-dim sampling and interpolation

Alexander Olevskii
Tel Aviv University, Israel
e-mail: olevskii@yahoo.com

I'll survey some classical and recent results on sampling/interpolation problems for functions with bounded spectrum.

Dynamical compact bodies in General Relativity

Todd Oliynyk
Monash University, Australia
e-mail: todd.oliynyk@monash.edu

The visible universe contains many different types of dynamical compact bodies, including asteroids, comets, planets, stars and even more exotic objects such as neutron stars. In spite of their fundamental importance to astrophysics and cosmology, there are currently very few analytical results available that apply to these dynamical bodies. In particular, even the most basic problem of establishing the (local) existence and uniqueness of solutions that represent gravitating compact bodies was, until very recently, an open problem in General Relativity (GR). In this talk, I will discuss this problem and pay particular attention to the case of elastic matter. After presenting some general background on the dynamics of compact bodies in GR, I will describe, in detail, the initial value formulation for the particular case of elastic matter and outline the analytic difficulties of the initial value problem for this system. I will then summarize recent results obtained in collaboration with Lars Anderson and Bernd Schmidt in which we establish the existence and uniqueness of solutions that represent gravitating dynamical elastic bodies. Time permitting, I will describe some open problems and promising directions for future work.

Harmonic measures of slit sides

Dmitry Prokhorov
Saratov State University, Russia
e-mail: ProkhorovDV@info.sgu.ru

We describe the geometry of solutions to the chordal Löwner differential equation which is based on the comparison of singular solutions and harmonic measures for the sides of a slit in domains generated by a driving term. In particular, it is proved that harmonic measures of two sides of a slit in the upper half-plane which is perpendicular to the real axis are asymptotically equal to each other.

Problems and Results in Nonlinear Analysis: An Update

Simeon Reich
The Technion - Israel Institute of Technology, Israel
e-mail: sreich@techunix.technion.ac.il

We intend to present an update regarding several open problems in Nonlinear Analysis which have been of recent research interest. These problems concern, for example, the generic method, infinite products of operators, as well as the asymptotic properties of holomorphic mappings.

Semi-linear structural damped waves

Michael Reissig
Technische Universität Bergakademie Freiberg, Germany
e-mail: reissig@mailserver.tu-freiberg.de

We study the Cauchy problem for the semi-linear structural damped wave equation with source term

$$u_{tt} - \triangle u + \mu(-\Delta)^{\sigma} u_t = f(u), \quad u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x),$$

with $\sigma \in (0,1]$ in space dimension $n \geq 2$ and with a positive constant μ. We are interested in the influence of σ on the critical exponent p_{crit} in $|f(u)| \approx |u|^p$. This critical exponent, the Fujita type exponent, is the threshold between global existence in time of small data solutions and blow-up behavior for some suitable range of p. To prove global existence in time of small data solutions we develop a WKB analysis for a parameter-dependent family of linear Cauchy problems. In this way, we are able to understand the source term as a small perturbation. The decay estimates of solutions to the linear model coincide with those ones for the semi-linear model. By using the test function method, we show the optimality of our results in the case $\sigma = 1/2$. The case $\sigma = 1/2$ is the threshold between parabolic like models for $\sigma \in (0,1/2)$ and hyperbolic like models $\sigma \in (1/2,1]$. This is joint work with Marcello D'Abbicco (Brescia).

D'Abbicco/Reissig, *Semi-linear structural damped waves*, 23 A4, Preprint 2012-05, Technical University Bergakademie Freiberg, ISSN 1433-9307.

Weighted norm inequalities for Radon transforms

Boris Rubin
Louisiana State University, USA
e-mail: borisr@math.lsu.edu

We obtain sharp inequalities for the k-plane transform, the "j-plane to k-plane" transform, and the corresponding dual transforms, acting on L^p spaces with a radial power weight. The operator norms are explicitly evaluated. Some generalizations and open problems are discussed.

Nodal intersections

Zeev Rudnick
Tel Aviv University, Israel
e-mail: rudnick@post.tau.ac.il

We study the fine structure of nodal lines for eigenfunctions of the Laplacian on a surface by examining the number of intersection of the nodal lines with a fixed reference curve. It is expected that in many cases the number of these intersections is bounded above by the wave number k (the square root of the eigenvalue). Very little is known about lower bounds.

For the flat torus, we prove the expected upper bound of k and give a lower bound of almost the same quality. To do so, we connect this problem to bounds on the L^p norms of the restriction of the eigenfunctions to the curve and to a problem in Number Theory. (joint work with Jean Bourgain).

Quantization on Lie groups

Michael Ruzhansky
Imperial College London, UK
e-mail: m.ruzhansky@imperial.ac.uk

We will give a survey of recent works on the global quantization theory on Lie groups, with different applications to harmonic analysis and PDEs.

On the continual Rubik's cube

Dmitry Ryabogin
Kent State University, USA
e-mail: ryabogin@math.kent.edu

Let f and g be two continuous functions on the unit sphere S^{n-1} in $\mathbb{R}^n$, $n \geq 3$, and let their restrictions to any one-dimensional great circle E coincide after some rotation ϕ_E of this circle: $f(\phi_E(\theta)) = g(\theta)\ \forall \theta \in E$. We prove that in this case $f(\theta) = g(\theta)$ or $f(\theta) = g(-\theta)$ for all $\theta \in S^{n-1}$. This answers the question posed by Richard Gardner and Vladimir Golubyatnikov.

On some generalized Hele-Shaw flows

Tatiana Savina
Ohio University, USA
e-mail: savin@ohio.edu

A Hele–Shaw cell is a device to study two-dimensional flows where fluids are sandwiched in a narrow gap between two parallel plates. We will discuss some flows in a Hele-Shaw cell including the cases where sink/source distribution has a linear support as well as the case of a cell with a time-dependent gap.

Geometric inequalities for hypersurfaces

Fernando Schwartz
University of Tennessee, USA
e-mail: fernando@math.utk.edu

I will begin this talk by recalling the classic inequalities of Alexandrov-Fenchel and Polya-Szego for convex surfaces of 3-dimensional Euclidean space. Then, I will present my joint work with Freire, which generalizes the inequalities -with rigidity- to both a larger class of

hypersurfaces and to arbitrary dimensions. I will conclude by mentioning some applications of the results, including an inequality for black holes.

Quantization of universal Teichmüller space

Armen Sergeev
Steklov Institute of Mathematics, Moscow, Russia
e-mail: sergeev@mi.ras.ru

Universal Teichmüller space $\mathcal{T}$ is the quotient of the group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of S^1 modulo Möbius transformations. In particular, this space contains the quotient $\mathcal{S}$ of the group $\mathrm{Diff}_+(S^1)$ of diffeomorphisms of S^1 modulo Möbius transformations. Both groups act naturally on Sobolev space $H := H_0^{1/2}(S^1, \mathbb{R})$.

Quantization problem for $\mathcal{T}$ and $\mathcal{S}$ arises in string theory where these spaces are considered as phase manifolds. To solve the problem for a given phase manifold means to fix a Lie algebra of functions (observables) on it and construct its irreducible representation in a Hilbert (quantization) space.

For $\mathcal{S}$, the algebra of observables is given by the Lie algebra $\mathrm{Vect}(S^1)$ of $\mathrm{Diff}_+(S^1)$. For quantization space we take the Fock space $F(H)$, associated with Sobolev space $H = H_0^{1/2}(S^1, \mathbb{R})$. Infinitesimal version of $\mathrm{Diff}_+(S^1)$-action on H generates an irreducible representation of $\mathrm{Vect}(S^1)$ in $F(H)$, yielding quantization of $\mathcal{S}$.

For $\mathcal{T}$ the situation is more subtle since $\mathrm{QS}(S^1)$-action on $\mathcal{T}$ is not smooth. So there is no classical Lie algebra associated to $\mathrm{QS}(S^1)$. However, we can define a quantum Lie algebra of observables $\mathrm{Der}^q(\mathrm{QS})$ generated by quantum differentials acting on $F(H)$. These differentials are given by integral operators $d^q h$ on H with kernels, given essentially by finite-difference derivatives of $h \in \mathrm{QS}(S^1)$.

On injectivity radius of local ring Q–homeomorphisms

Yevgen Sevostyanov
Institute of Applied Mathematics and Mechanics, Ukraine
e-mail: brusin2006@rambler.ru

Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \mathbb{R}^n$ be a continuous mapping. Recall a mapping $f : D \to \mathbb{R}^n$ is said to be *local homeomorphism* in D, if for every $x_0 \in D$ there is $\delta > 0$ such that $f|_{B(x_0,\delta)}$ is homeomorphic; here $B(x_0, \delta)$ denotes the ball in $\mathbb{R}^n$ centered at x_0 with radius δ.

Denote by $S(x_0, r_1)$ and $S(x_0, r_2)$ the corresponding boundaries of the spherical ring $A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}$ and let Γ be a family of paths γ in $\mathbb{R}^n$ which join $S(x_0, r_1)$ and $S(x_0, r_2)$ in $A(x_0, r_1, r_2)$. Given a (Lebesgue) measurable function $Q : D \to [0, \infty]$, a mapping $f : D \to \mathbb{R}^n$ is called *ring Q–mapping at a point $x_0 \in D$* if the conformal modulus satisfies the following inequality

$$M\left(f(\Gamma(S(x_0, r_1), S(x_0, r_2), A(x_0, r_1, r_2)))\right) \leq \int\limits_{A(x_0,r_1,r_2)} Q(x) \cdot \eta^n(|x - x_0|)\, dm(x)$$

for any $A(x_0, r_1, r_2)$, $0 < r_1 < r_2 < r_0 = \operatorname{dist}(x_0, \partial D)$, and for every Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r) dr \geq 1$.

We also denote by $q_{x_0}(r)$ the integral average of $Q(x)$ over $|x - x_0| = r$, i.e.

$$q_{x_0}(r) := \frac{1}{\omega_{n-1} r^{n-1}} \int\limits_{|x-x_0|=r} Q(x)\, dS,$$

where ω_{n-1} is the area of $\mathbb{B}^n = B(0, 1)$.

The main result of the talk is the following

Theorem. *Let $f : \mathbb{B}^n \to \mathbb{R}^n$ be a local ring Q–homeomorphism at the origin with $Q \in L^1_{\rm loc}(\mathbb{B}^n)$ providing*

$$\int\limits_0^1 \frac{dt}{t q_0^{1/(n-1)}(t)} = \infty\,. \tag{3}$$

Then f is injective in the ball $B\,(0, \delta)$, where $\delta > 0$ depends only on n and Q.

Note that equality (3) provides a necessary condition for injectivity of mapping, i.e. for any $\delta > 0$ and every function $Q \in L^1_{\rm loc}(\mathbb{B}^n)$ satisfying

$$\int\limits_0^1 \frac{dt}{t q_0^{1/(n-1)}(t)} < \infty,$$

there exists a local ring Q–homeomorphism $f = f_Q : \mathbb{B}^n \to \mathbb{R}^n$ at the point $x_0 = 0$ which is not injective in $B(0, \delta)$.

Localization of singularities of solutions to semi-linear parabolic and elliptic equations with degenerate absorption potential

Andrey E. Shishkov
Institute of Applied Mathematics and Mechanics, Ukraine
e-mail: shishkov@iamm.ac.donetsk.ua

We study existence and the limit behaviour as $k \to \infty$ of solutions u_k to the Cauchy problem:

$$u_t - \Delta u + h u = 0 \quad \text{in} \quad \mathbb{R}^N \times (0, \infty), \quad u(x, 0) = k\delta_0(x), \quad p > 1, \quad N \geqslant 1,$$

where $\delta_0(x)$ is Dirac measure, nonnegative continuous function $h(x, t) = 0$ on some smooth manifold Γ with $(0, 0) \in \Gamma$. Particularly, if $h = h(|x|) = |x|^\beta$, $\beta > 0$, then by arbitrary $\lambda : 1 < \lambda < \lambda_{\rm cr} := 1 + \frac{2+\beta}{N}$ for each $k \in \mathbb{N}$ there exists "fundamental" solution u_k and limiting solution u_∞ is an explicit very singular (more singular than u_k) solution with point singularity at $(0, 0)$. In the opposite case, when $\lambda \geqslant \lambda_{\rm cr}$, the mentioned problem has no solution for any $k \in \mathbb{N}$ [1].

Strong degeneration of potential h yields the following new phenomenon. By $k \to \infty$, point singularity of solutions u_k may propagate on all manifold Γ and, as a result, u_∞ turns

into solution with nonlocalized singularity set Γ (for example, "razor blade" solution). For some model manifolds (partiacularly, $\Gamma_1 = \{0, t\}$, $\Gamma_2 = \{x, 0\}$) we found sharp necessary and sufficient condition (criterium) on the flatness of h near to Γ, guaranteeng propagation or nonpropagation of singularity set on Γ. We investigate mentioned phenomenon for different classes of quasilinear parabolic diffusion-absarption type equation (porous medium, evolution p-Laplace) with degenerate absorption potential. A stationary (elliptic) version of the mentioned theory of propagation-nonpropagation of singularities of very singular solutions (see [2], [3]) will be discussed too. Results of joint works with Laurent Veron, Moshe Marcus.

References

[1] Shishkov A., Veron L. Singular solutions of some nonlinear parabolic equations with spatially inhomogeneous absorption, *Calc. Var. Part. Differ. Equat.,* **33** (2008), p. 343–375.

[2] Shishkov A., Veron L. Diffusion versus absorption in semi-linear elliptic equations, *J. Math. Anal. Appl.,* **352** (2009), p. 206–217.

[3] Murcus M., Shishkov A. Fading absorption in non-linear elliptic equations, *Ann. I. H. Poincaré* – AN (2012), http://dx.doi.org/10.1016/j. anihpe. 2012.08.002.

On the number of components of zero sets of smooth random functions of several real variables

Mikhail Sodin
Tel Aviv University, Israel
e-mail: sodin@post.tau.ac.il

In a joint work with Fedor Nazarov, we study the number of components of zero sets of smooth Gaussian random functions of several real variables. The primary examples are various Gaussian ensembles of real-valued polynomials (algebraic or trigonometric) of large degree on the sphere or torus and translation-invariant smooth Gaussian functions on the Euclidean space restricted to domains of large volume. This can be viewed as a statistical version of (the first part) of Hilbert's 16th problem.

The space of positive Lagrangian submanifolds

Jake Solomon
Hebrew University, Jerusalem, Israel
e-mail: jake@math.huji.ac.il

A Lagrangian submanifold of a Calabi–Yau manifold is called positive if the real part of the holomorphic volume form restricted to it is positive. A Hamiltonian isotopy class of positive Lagrangian submanifolds admits a Riemannian metric with non-positive curvature. Its universal cover admits a functional, with critical points special Lagrangians, that is strictly convex with respect to the metric. If time permits, I'll explain how mirror symmetry relates the metric and functional to the infinite dimensional symplectic reduction picture of Atiyah, Bott, and Donaldson in the context of the Kobayashi–Hitchin correspondence.

Minimization of area: Iceberg-type problems in the plane

Alexander Solynin
Texas Tech University, USA
e-mail: alex.solynin@ttu.edu

We will discuss several extremal problems concerning area and some other characteristics of two-dimensional configurations. The study of these problems began forty years ago with works of Harold Shapiro and Dov Aharonov on the minimal area problem for normalized univalent functions. In this talk, I will focus on problems for "partially visible sets", when certain characteristics of a portion of a given set are known while some other characteristics of the complementary portion of the set are have to be estimated. These so-called "iceberg-type problems" were discussed in our recent publications joint with R. Barnard, K. Pearce and M. Lochman.

Recent progress in smoothing estimates for Schrödinger equations

Mitsuru Sugimoto
Nagoya University, Japan
e-mail: sugimoto@math.nagoya-u.ac.jp

It is well known that solutions to initial data problem for Schrödinger equations have an extra gain of regularity by taking an integral mean in the time variable. We will discuss how some methods of comparison and canonical transformation work in showing such smoothing estimates not only for Schrödinger equation but also for other various types of evolution equation. Some application to nonlinear problems, the optimal constants and extremising initial data, and a new method of spectral comparison will be also presented.

Weakly hyperbolic equations with nonanalytic coefficients

Giovanni Taglialatela
University of Bari, Italy
e-mail: giovanni.taglialatela@uniba.it

The Cauchy problem for linear weakly hyperbolic equations with C-infinity coefficients has been extensively studied in the literature.

On the other side, Colombini et al. (Ann. S.N.S. 1979) and Colombini et al. (Ann. S.N.S. 1983) considered second order wave type operators and established the relation between the C^k regularity of the coefficients and the spaces in which the Cauchy problem is well-posed. However, when trying to generalize these results to wider classes of equations, many problems arise. In this talk, I will present some results concerning third order equations obtained with E. Jannelli.

An extremal problem related to analytic continuation

Nikolai Tarkhanov
Universität Potsdam, Germany
e-mail: tarkhanov@math.uni-potsdam.de

We show that the usual variational formulation of the problem of analytic continuation from an arc on the boundary of a plane domain does not lead to a relaxation of this overdetermined problem. To attain such a relaxation, we bound the domain of the functional, thus changing the Euler equations. An approximate solution can be constructed, e.g., by the classical Ritz method.

Extremal problems as limit points in deformation theory

Razvan Teodorescu
University of South Florida, USA
e-mail: razvan@usf.edu

Over-determined boundary-value problems in the complex plane share several similarities to singular free-boundary problems, from the absence of a variational formulation to a rich symmetry group leaving the solution invariant.

In this talk, we show that both classes of problems can be formulated as limit points in deformation theory, which explains the similarities and provides new methods for finding solutions.

Wiener type theorems on Fourier series with positive coefficients

Sergey Tikhonov
ICREA and CRM, Spain
e-mail: tikhonov.work@gmail.com

We discuss recent achievements on Wiener's problem for the trigonometric Fourier series and the Fourier–Jacobi series with non-negative coefficients. Related Paley's theorem will be discussed.

On the loss of regularity for the three-dimensional Euler equations

Edriss S. Titi
Weizmann Institute of Science, Israel, The University of California - Irvine
e-mail: etiti@math.uci.edu

A basic example of shear flow was introduced by DiPerna and Majda to study the weak limit of oscillatory solutions of the Euler equations of incompressible ideal fluids. In particular, they proved by means of this example that weak limit of solutions of Euler equations may, in some cases, fail to be a solution of Euler equations. We use this shear flow example to provide non-generic, yet nontrivial, examples concerning the immediate loss of smoothness and ill-posedness of solutions of the three-dimensional Euler equations for initial

data that do not belong to $C^{1,\alpha}$. Moreover, we show by means of this shear flow example the existence of weak solutions for the three-dimensional Euler equations with vorticity that is having a nontrivial density concentrated on non-smooth surface. This is very different from what has been proven for the two-dimensional Kelvin-Helmholtz problem where a minimal regularity implies the real analyticity of the interface. Eventually, we use this shear flow to provide explicit examples of non-regular solutions of the three-dimensional Euler equations that conserve the energy, an issue which is related to the Onsager conjecture. In addition, we will use this shear flow to provide a nontrivial example for the use of vanishing viscosity limit, of the Navier-Stokes solutions, as a selection principle for uniqueness of weak solutions of the 3D Euler equations. This is a joint work with Claude Bardos.

Minimizing discrete energy on the sphere

Alexander Tumanov
University of Illinois, USA
e-mail: tumanov@illinois.edu

Consider N points on the unit 2-sphere. The energy of the interaction of two points on the sphere is a function $f(r)$ of the distance r between the points. The total energy E of N points is the sum of the pairwise energies. The problem is how to place the points on the sphere to minimize the energy E. This problem is a subject of hundreds of publications. For the Coulomb potential $f(r) = 1/r$, the problem has a significance in physics and goes back to Thompson (1904). The results for $N < 5$ are well known. We focus on the case $N = 5$ which turns out to be difficult. In this case, the following results have been obtained. Dragnev, Legg, and Townsend (2002) give a solution of the problem for $f(r) = -\log r$ known as Whyte's problem. Hou and Shao (2009) give a rigorous computer-aided solution for $f(r) = -r$, for which the problem is well-known in discrete geometry. Schwartz (2010) gives a rigorous computer-aided solution of Thomson's problem. The results of Hou and Shao and Schwartz involve massive calculations that require a computer. We give a solution for biquadratic potentials avoiding heavy calculations.

Conformal composition operators and Brennan's conjecture

Alexander Ukhlov
Ben-Gurion University of the Negev, Israel
e-mail: ukhlov@math.bgu.ac.il

We study composition operators on Sobolev spaces generated by conformal mappings of plane Euclidean domains $\Omega, \Omega' \subset \mathbb{R}^2$ in connection with Brennan's conjecture. Brennan's conjecture states integrability of the complex derivative φ' of a plane conformal mapping $\varphi : \Omega \to \mathbb{D}$, $\mathbb{D}$ is the unit disc, in the power $4/3 < s < 4$. We prove that Brennan's conjecture holds if and only if φ generates by the composition rule $\varphi^*(f) = f \circ \varphi$, $f \in L^1_p(\mathbb{D})$, $2 < p < \infty$, a bounded composition operator

$$\varphi^* : L^1_p(\mathbb{D}) \to L^1_q(\Omega), \quad q = ps/(p + s - 2).$$

This result has applications in the weighted Sobolev type embeddings and degenerate elliptic boundary value problems.

Stochastic Loewner-Kufarev evolution with a random Herglotz field

Alexander Vasil'ev
Universitetet i Bergen, Norway
e-mail: alexander.vasiliev@math.uib.no

We work with a stochastic version of the general version of the Loewner evolution. We study the case of the Herglotz vector field when the attractive point moves along the boundary of the unit disc. In the stochastic case, it realizes the Brownian motion on the circle. Relations to SLE are shown.

Spectral theory of first order elliptic systems

Dmitri Vassiliev
University College London, UK
e-mail: D.Vassiliev@ucl.ac.uk

We work on a compact manifold without boundary and consider the spectral problem for an elliptic self-adjoint first order system of (pseudo)differential equations. The eigenvalues of the principal symbol are assumed to be simple but no assumptions are made on their sign. So the operator is not necessarily semi-bounded. The objective is to derive a two-term asymptotic formula for the counting function (number of eigenvalues between zero and a positive lambda) as lambda tends to plus infinity.

The author has recently discovered [1] that all previous publications on first order systems give formulae for the second asymptotic coefficient that are either incorrect or incomplete (i.e. an algorithm for the calculation of the second asymptotic coefficient rather than an explicit formula). The aim of the talk is to present the correct formula for the second asymptotic coefficient and discuss its geometric meaning.

References

[1] Preprint arXiv:1208.6015. To appear in Journal of Spectral Theory.

Briançon–Skoda theorem for quotient ring

Alekos Vidras
University of Cyprus, Cyprus
e-mail: msvidras@ucy.ac.c

Let $\mathcal{I}$ be any ideal local regular ring $\mathcal{O}_n$ of germs of analytic functions in the neighborhood of the origin $0 \in \mathbb{C}^n$ generated by m elements. Then Briançon-Skoda theorem states that $\forall k \in \mathbb{N}^*$ one has that $\overline{\mathcal{I}^{\mu+k-1}} \subset \mathcal{I}^k$, where $\mu = \min\{n, m\}$. The overline denotes the algebraic closure of an ideal. We prove similar result for ideals contained in the the sheaf of quotient rings $\mathcal{O}_{\mathcal{V},0} = \mathcal{O}_n/\mathcal{I}_{\mathcal{V},0}$, where $\mathcal{V}$ is a purely M-codimensional analytic subset about the origin of $\mathbb{C}^n$ and $\mathcal{I}_{\mathcal{V},0}$ be its sheaf of radical ideals. The result is a joint work with A. Yger.

On mappings with bounded codistortion

Sergey K. Vodopyanov
Sobolev Institute of mathematics, Novosibirsk, Russia
e-mail: vodopis@math.nsc.ru

We suppose to introduce a new class of mappings in the Euclidean space of the dimension $n > 2$ and show its properties. The new class consists of mappings $f : \Omega \to \mathbb{R}^n$ belonging to $W^1_{n,\mathrm{loc}}(\Omega)$ and havings the bounded codistortion:

$$|\operatorname{adj} Df(x)|^{\frac{n}{n-1}} \le KJ(x,f) \quad \text{a. e. in} \quad \Omega.$$

Here $Df(x)$ is the differential of f at a point $x \in \Omega$, $J(x,f) = \det Df(x)$, and the adjoint matrix $\operatorname{adj} Df(x)$ is defined by the condition $Df(x)\operatorname{adj} Df(x) = I \cdot J(x,f)$ at the points $x \in \Omega$ where $J(x,f) \neq 0$ and by the continuity in the others.

This research was carried out with the partial support of the Russian Foundation for Basic Research (grant no. 11-01-00819).

References

[1] S. K. Vodopyanov, Regularity of mappings inverse to Sobolev mappings, *Sbornik: Mathematics* **203**: 10, 2012, pp. 1383–1410.

[2] S. K. Vodopyanov, On regularity of Poletskys function under weak analytic assumptions of the given mapping, *Dokl. Math.* 2013 (to appear).

The Penrose Inequality with charge

Gilbert Weinstein
Monash University, Australia
e-mail: gilbert.weinstein@gmail.com

I will report on progress made towards a proof of the Penrose inequality with charge

$$R \le m + \sqrt{m^2 - Q^2}$$

where m is the ADM mass, R the area radius of the outermost horizon, and Q the total charge of an initial data set for the Einstein-Maxwell equations. The proof uses a version of H. Bray's conformal flow, modified to accommodate the charge. This is joint work with M. Khuri and S. Yamada.

The Klein–Gordon Equation in the curved spacetime

Karen Yagdjian
University of Texas - Pan American, USA
e-mail: yagdjian@utpa.edu

In this talk we present some results on the global existence of the small data solutions of the Cauchy problem for the system of semilinear Klein–Gordon equations in the de Sitter spacetime. The existence is proved under assumption that the eigenvalues of the mass matrix are outside of some open bounded interval. The relations to the Higuchi bound and Huygens' Principle are revealed. The asymptotic for the solution of system of semilinear equations is obtained. In the proof, we use the explicit representation of the solutions of linear equations and the L_p-L_q estimates.

Generalized Remez inequality for $(s;p)$-valent functions

Yosef Yomdin
Weizmann Institute of Science, Rehovot, Israel
e-mail: yosef.yomdin@weizmann.ac.il

The classical Remez inequality bounds the maximum of the absolute value of a polynomial $P(x)$ of degree d on $[-1,1]$ through the maximum of its absolute value on any subset Z of positive measure in $[-1,1]$. It is known that the Lebesgue measure in the Remez inequality can be replaced by a certain geometric invariant $s(Z)$ which can be effectively estimated in terms of the metric entropy of Z and which may be nonzero for discrete and even finite sets Z. In this talk, we discuss an essentially sharp Remez-type inequality for complex polynomials of one variable, introducing metric invariant $c(Z)$ for an arbitrary bounded subset Z of the complex plane. This invariant translates the classical Cartan lemma into metric language. Next we introduce (s,p)-valent functions, which provide a natural generalization of p-valent ones. We prove a distortion theorem for such functions, comparing them with polynomials sharing their zeroes. On this base, we extend to (s,p)-valent functions our polynomial Remez-type inequality.

Automorphism groups of affine varieties and their Lie algebras

Mikhail Zaidenberg
Institut Fourier, Grenoble, France
e-mail: zaidenbe@ujf-grenoble.fr

Given a projective algebraic variety, the identity component of its automorphism group is known to be an algebraic group. In contrast, for an affine algebraic variety this group can be infinite dimensional, and its structure can occur to be quite mysterious. What is the Lie algebra of this group? We will survey this subject, both some classical results and a very recent development. We will provide some concrete examples (Gizatullin surfaces, configuration spaces, affine cones over del Pezzo surfaces, over Fano threefolds, etc.) This is a report on a joint work with Ivan Arzhantsev, Hubert Flenner, Shulim Kaliman, Takashi

Kishimoto, Hanspeter Kraft, Frank Kutzschebauch, Karine Kuyumzhiyan, Vladimir Lin, Alexandre Perepechko, and Yuri Prokhorov.

Internal capacity characteristics of domains in several complex variables

Vyacheslav Zakharyuta
Sabanci University, Istanbul, Turkey
e-mail: zaha@sabancinuiv.edu

My talk is devoted to internal capacity characteristics of domains in several complex variable, relative to a point. Our main goal is to study internal Chebyshev constants and transfinite diameters of domains and their boundaries in the spirit of author's article (Math USSR Sbornik 25, 1975), where those characteristics were considered for compact sets. Some results are closely related to results on Reiffen pseudometrics and internal directional analytic capacities (Jarnicki–Pflug and Nivoche).

Quantization of Whitney problems

Nahum Zobin
College of William and Mary, USA
e-mail: nxzobi@wm.edu

Quantitative versions of Whitney problems require construction of functions with prescribed values on a given finite subset $E \subset \mathbb{R}^n$, which minimize a preferred functional norm. After solving this minimization problem, we want to compute some natural functionals of the minimizer, e.g., its values at other points. Quantization is an art of replacing a minimization problem by a problem of computing certain amplitudes (similar to expected values) for a system where the preferred functional norm is treated as an action functional. There is an interesting connection between the computation of amplitudes (which are represented as functional integrals) and computations of convolutions of functions on some important unipotent Lie groups, similar to the Heisenberg–Weyl groups.