ABSTRACTS

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Minimal area problems and quadrature domains

I will recall some of the background needed for investigating minimal area problems with side conditions. I will then explain how Harold Shapiro and myself were led, while working on these problems, to study the (generalized) Schwarz function and quadrature domains in the complex plane. Also I will talk about some recent developments in symmetrization theory that enabled Alexander Solynin, Harold Shapiro and myself to settle an old specific open problem in this area. The general minimal area problem cannot be attacked by the method of symmetrization and is still wide open.

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The classical theorem of Rogosinski on partial sums of power series, its generalizations and applications

The well-known theorem of Rogosinski asserts that if the modulus of the sum of a power series is less than 1 in the open unit disk: $\left|\sum_{n=0}^{\infty} a_n z^n\right| < 1,\ |z| < 1$, then all its partial sums are less then 1 in the disk of radius 1/2:

$$\left|\sum_{n=0}^{k} a_n z^n\right| < 1,\ |z| < \frac{1}{2}\,,$$

and this radius is sharp.

We present a generalization of this theorem for holomorphic mappings of the open unit ball into an arbitrary convex domain. Other multidimensional analogs of Rogosinski's theorem, as well as some applications to dynamic systems, are considered.

The results obtained are joint work with M. Elin and D. Shoikhet.

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The dyadic parameterization of curves (with F. D. Lesley and V. I. Rotar)

A dyadic parametrization of curves is introduced. This enables connections to be made between the tangential or spiralling properties of curves (usually non-rectifiable) and classical theorems of probability. This is very useful for "constructing" examples.

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Some characteristics of asymptotic behavior of subharmonic and entire functions and their independence

It is known [1, Ch.3, Th.18] that using any *total* family of asymptotic characteristics, we can check that an entire function of finite order is a function of *completely regular growth* [2, Ch. 2, 3]. However, even total families do not allow one to check whether a function has an *H-multiplicator* [3] for a strictly ρ-trigonometrically convex function H.

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Normal families and exceptional values of derivatives

A classical result of Hayman says that if f is meromorphic in the plane and f and $f^{(k)} - 1$ have no zeros for some $k \geq 1$, then f is constant. The corresponding normality result according to Bloch's heuristic principle is due to Gu. We shall discuss some generalizations of these results where instead of assuming that f and $f^{(k)} - 1$ have no zeros we only assume that the zeros of these functions are of sufficiently high multiplicity. We also discuss the normality of the family of the logarithmic derivatives f'/f under the hypotheses that f and $f^{(k)}$ have no zeros.

The results are joint work with J. K. Langley.

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Planar Harmonic Mappings

We review some recent results in the field in two different directions: univalent and multivalent mappings with given dilatations and zeros of polynomials.

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Sub-Riemannian geometry on a step 2k sub-Riemannian manifold

In this talk, we discuss the behavior of sub-Riemannian geodesics on a certain step 2k sub-Riemannian manifold. We compute the length of the sub-Riemannian geodesics between the origin and any other point. We also characterize the number of sub-Riemannian geodesics between the origin and any other point.

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Simple curves in $\mathbb{R}^n$ and Ahlfors' Schwarzian derivative

We derive sharp injectivity criteria for mappings $f : (-1, 1) \to \mathbb{R}^n$ in terms of Ahlfors' definition of the Schwarzian derivative for such mappings. In addition, we study the issues of continuous extendibility of f to $[-1, 1]$ and of extremal behavior, establishing results similar in spirit to theorems of Gehring and Pommerenke.

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Moduli of continuity and K-divisibility of K-functionals

Most of this talk will be a historical survey. Its main aim is to draw attention to the important and by now classical K-divisibility theorem of Yuri Brudnyi and Natan Krugljak. Apart from having several significant consequences in the theory of interpolation spaces, this theorem has implications well beyond interpolation theory, e.g. in connection with moduli of continuity. It seems to me that the full potential of these implications has yet to be realized. Various results about estimates for K-divisibility constants will also be mentioned, including very recent joint work with Yacin Ameur and some earlier results obtained with Uri Keich.

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Direct and inverse spectral problem for generalized Jacobi matrices

A new class of generalized Jacobi matrices is introduced. Every generalized Nevanlinna function $m(z)$, which is a solution of an indefinite moment problem is proved to be the m-function of a unique generalized Jacobi matrix. The method we use is based on the step-by-step Schur process of solving the indefinite moment problem. As a corollary we obtain the following analog of the Stone theorem: every cyclic self-adjoint operator in a Pontryagin space is unitary equivalent to some generalized Jacobi matrix. Explicit formulas which reconsruct the finite generalized Jacobi matrix H from the spectral data via the trace formulas are found. The results were obtained in a joint work with M. Derevjagin.

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Riccati equations and bitangential interpolation problems

The aim of this talk is to explain the useful role of Riccati equations in the analysis of bitangential interpolation problems with singular Pick matrices. The talk will be expository and will assume little beyond some elementary complex analysis.

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The solutions of some Diophantine equations

Questions about the representation of positive integers as a sum of 2 or 4 squares go back at least to Fermat and Legendre. Jacobi however solved the quatititative result and gave a formula for the number of solutions of the Diophantine equations $x^2 + y^2 = n,\ x^2 + y^2 + z^2 + w^2 = n$. In the talk I shall show how the number of solutions of the former equation gives us a formula for the number of solutions of the latter. The tool will be the theory of logarithmic derivatives of theta functions. As further examples we shall give formulas for the number of solutions to the Diophantine equations: $x^2 + y^2 + 2z^2 + 2w^2 = n,\ x^2 + y^2 + 3z^2 + 3w^2 = n.$

As in the case of Jacobi, the formula will involve a variant of the classical sigma function which sums the divisors of a positive integer.

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The image of the sum of slit mappings

Consider a finitely connected plane domain. There is a normalized, analytic mapping of the domain onto the complement of slits in assigned directions. There is another normalized, analytic mapping onto the complement of slits orthogonal to those of the first mapping. The sum of these mappings is a mapping of the original domain onto the complement of convex sets. More information is sought about the image domain. Also, a formula is sought for the normalized slit mapping with the assigned angles. This last slit mapping has as its domain, the image of the original pair of slit mappings.

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Billiards in an ellipse

The billiard problem was introduced by Birkhoff in his investigations of Hamiltonian systems. For elliptical tables, the problem is integrable and is related to a classical geometric problem of Poncelet. Using notions from Riemann surfaces, we obtain the invariant measure associated with this problem. This is in turn used to obtain a complete description of all possible motions for billiards in an ellipse.

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Variational methods for first-order univalence criteria

A first-order univalence criterion for a domain $D \subset \mathbf{C}$ is a condition of the form $f'(D) \subset R$ which implies that f is univalent in D; such a criterion is said to be (weakly) sharp if for all $\epsilon > 0$ there are nonunivalent g on D for which $g'(D)$ is contained in the ϵ-neighborhood of R. Trivial arguments show that for a given D vast numbers of such sharp criteria exist; finding them is, however, a difficult matter. Indeed, the 27-year-old question of John ([1], [3]) about the largest annulus $A = \{z : 1 < |z| < \rho\}$ such that $f'(\Delta) \subset A$ is a univalence criterion for the unit disk Δ, remains unanswered. Under minimal assumptions about D and R the discovery of such sharp criteria can be reduced to the analysis of corresponding extremal functions f_0, i.e., functions for which $f_0'(D) \subset R$ and $f_0(a) = f_0(b)$, for two distinct points a, $b \in \partial D$. We have conjectured that if D is smoothly bounded and simply connected and R is smoothly bounded, simply or doubly connected, and contained in an annulus $0 < r_1 < |z| < r_2$, then for any such extremal f_0, $f_0' = T_R(B(T_D^{-1})))$, where T_D and T_R are canonical mappings of the unit disk onto D and (the universal covering surface of) R, respectively, and B is a finite Blaschke product. In [2] we established this for the case of ring domains R bounded by two curves strictly star-shaped with respect to 0 and drew from this a number of qualitative conclusions about the corresponding criteria. In that paper we used a variational procedure based on the representation of $f_0(z)$ as $G(h_0(z))$, where G is a canonical mapping of the strip $S_0 = \{z : |\Re z| < 1\}$ onto the universal covering surface of R, and $|\Re h_0(z)| \leq 1$ in D. In this talk we shall discuss various aspects of our previous work on first-order univalence criteria as well as recent improvements, involving the analysis of the second variation and a more general variational method employing strips with vertical slits in addition to S_0, which allow one to extend the results of [2] to many simply connected R for which the methods of that paper were not sufficient.

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Maximal ideal spaces of invariant algebras

Let P be a homogeneous space of a compact Lie group G, A be a closed G-invariant subalgebra of the Banach algebra C(P), and M be its maximal ideal space. If A is generated by a finite dimensional invariant subspace then M can be identified with the polynomially convex hull of an orbit in a finite dimensional linear space. The talking concerns the problem of a determination of M. There is a satisfactory solution for two opposite cases. 1) If P=G with G acting by left and right then M is a semigroup and the desired description exists. 2) There are necessary and sufficient conditions for an orbit to be polynomially convex (joint result with I. Latypov). For the generic case there are partial results.

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Analyticity on circles

Let U be the union of a family S of circles in the complex plane and let f be a continuous function on U which extends holomorphically from each circle C in S to the disc bounded by C. When does it follow that f is holomorphic in the interior of U? We will show how some tools from several complex variables can be used to deal with this problem.

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On a connection between the number of poles of a meromorphic function and the number of zeros its derivatives

Let f be a meromorphic in finite plane function. We prove an estimate of the function $N(r, f)$ from above by using a sum of the functions $N(r, 1/f^k)$ for some set of $k \in \mathbb{N}$ without any restriction of mutiplicity of poles.

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Homeomorphism with mean integral dilatations

One of the basic properties characterizing quasiconformal mappings is the quasi-invariance of the n-module of the families of separating surfaces. We consider more general inequalities. Let G and G^* be two bounded domains in $\mathbb{R}^n$, $n \geq 2$. For a ring domain $D \subset \mathbb{R}^n$, denote by Σ_D the family of all compact piecewise smooth $(n-1)$-dimensional surfaces Σ which separate the boundary components of D. Let Φ be a finite nonnegative function in G defined for open subsets E of G such that $\sum_{k=1}^{m} \Phi(E_k) \leqslant \Phi(E)$ for any finite collection $\{E_k\}_{k=1}^{m}$ of nonintersecting open sets $E_k \subset E$. We denote the class of all such set functions Φ by F. Fix the numbers $\alpha, \beta, \gamma, \delta$ satisfying

$$n-1 \leq \alpha < \beta < \infty, \quad n-1 \leq \gamma < \delta < \infty,$$

and assume that there exists a nonempty family of homeomorphisms $f : G \to G^*$ such that there are two set functions $\Phi, \Psi \in F$ not depending on f such that for each ring domain $D \subset G$ the inequalities

$$M_{\alpha}^{\beta}(\Sigma_D^*) \leq \Phi^{\beta-\alpha}(D) M_{\beta}^{\alpha}(\Sigma_D),$$

$$M_{\gamma}^{\delta}(\Sigma_D) \leq \Psi^{\delta-\gamma}(D) M_{\delta}^{\gamma}(\Sigma_D^*),$$

hold. Here $M_p(\Sigma_D)$ is the p-module of Σ_D and $\Sigma_D^* = f(\Sigma_D)$. The class of such homeomorphisms is denoted by $M\mathcal{S}(G)$ (in fact, it also depends on $\alpha, \beta, \gamma, \delta$). Our main result on the differential and geometric properties of homeomorphisms with mean quasiconformal dilatations is

Theorem 1. *Let* $n-1 < \alpha < \beta \leq n$ *and* $n-1 < \gamma < \delta \leq n$ *or* $n \leq \alpha < \beta < (n-1)^2/(n-2)$ *and* $n \leq \gamma < \delta < (n-1)^2/(n-2)$. *Then every mapping* $f \in M\mathcal{S}(G)$ *possesses the following properties: (a)* f *and* f^{-1} *are ACL (absolutely continuous on lines); (b)* $f \in W_{1,loc}^{p}(G)$ *with* $p = \max\big(\gamma, \beta/(\beta-n+1)\big)$*; (c)* $f^{-1} \in W_{1,loc}^{q}(G^*)$, $q = \max\big(\alpha, \delta/(\delta-n+1)\big)$.

These bounds for the degrees of integrability are sharp. This result is strengthened for some special subclasses of mappings. For example, when f is quasiisometric, quasiconformal, etc., (in the last case, we obtain a new characterization of quasiconformality).

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Evolution families of analytic functions

The dynamics of various physical processes are described by means of the composition of analytic functions. As a case in point, we mention stochastic branching process theory, which studies evolution of a population or development of paste-like matter in discrete or continuous time. On the assumption of homogeneity, the dynamic of a process with discrete time is described by iterates of an analytic function. A homogeneous process with continuous time is connected with the corresponding one-parameter semigroup of analytic functions. The question of embedding analytic functions into continuous semigroups of fractional iterates arises in the study of embedding homogeneous processes with discrete time into homogeneous processes with continuous time. Recently, the problem of fractional iteration for a general semigroup $\mathfrak{P}$ composed of holomorphic mappings f of the unit disk $\mathbb{D} = \{z \in \mathbb{C} : \ |z| < 1\}$ into itself was solved in various terms in [1]. We say that $f \in \mathfrak{P}$ is embeddable if there exists a family $\{f^t\}_{t\geq 0}$ in $\mathfrak{P}$ such that $f^0(z) = z$, $f^1(z) = f(z)$, $f^{t+s}(z) = f^t \circ f^s(z)$ for $s, t \geq 0$, and $f^t(z) \to z$ locally uniformly with respect to $z \in \mathbb{D}$ as $t \to 0$. In other words, $t \to f^t$ is a one-parameter semigroup in $\mathfrak{P}$. The following theorem shows which restrictions on f are followed to be embeddable.

Theorem. *Let $f(z) = c_1 z + c_2 z^2 + \ldots \in \mathfrak{P}$ be embeddable and different from a Möbius transformation. Then the point (c_1, c_2) belongs to the set:*

$$\{(w_1, w_2) \in \mathbb{C}^2 : \quad w_1 = e^{-\xi}, \quad w_2 = 2w_1(1 - w_1)\frac{\mathrm{Re}\,\xi}{|\xi|}\eta, \quad \mathrm{Re}\xi > 0, \quad |\eta| \leq 1\}.$$

It may be noted that for an arbitrary $f(z) = c_1 z + c_2 z^2 + \ldots$ in $\mathfrak{P}$ the coefficients c_1, c_2 satisfy the following inequalities

$$|c_1| \leq 1, \qquad |c_2| \leq 1 - |c_1|^2.$$

Let $\mathfrak{G}$ be a semigroup of analytic functions. We say that a subset $\{w_{t,s} : 0 \leq s \leq t \leq T\}$ of $\mathfrak{G}$ is an evolution family in $\mathfrak{G}$ if the following conditions hold:

(i) $w_{t,t}(z) = z$ for $0 \leq t \leq T$;

(ii) $w_{t,s}(z) = w_{t,r} \circ w_{r,s}(z)$ for $0 \leq s \leq r \leq t \leq T$;

(iii) $w_{t,s}(z) \to z$ locally uniformly with respect to $z \in \mathbb{D}$ as $t, s \to r$.

Evolution families naturally arise in the study of nonhomogeneous processes with continuous time. In contrast to one-parameter semigroups, the property of differentiability for an evolution family is more complicated [2]. We study conditions which guarantee the differentiability of evolution families. The point is that these families could be described by an evolution equation.

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Painleve differential system in complex plane

We discuss the Painleve property and Backlund transformations for the nonlinear ordinary differential systems and review several important general properties of the Painleve systems, such as character of singularities, representation of solutions, special solutions and the number of poles of solutions.

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Zeros of solutions to the functional equations $\sum_{j=1}^{m} a_j(z)f(c^j z) = Q(z)$

We discuss zeros of solutions to the functional equations

$$\sum_{j=1}^{m} a_j(z)f(c^j z) = Q(z).$$

Here $0 < |c| < 1$ and Q and the a_j are polynomials.

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Growth estimates for certain analytic functions

We discuss growth estimates for analytic functions in the unit disk such that the function and some of its derivatives omit certain values. This is joint work with W.K. Hayman.

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On the possiblity of continuation of functions from a part of the boundary of a domain to the whole domain as solutions of the Moisil-Theodoresco system of equations

The problem of describing the functions given on a part of the boundary of a domain which can be holomorphically continued into the domain has been thoroughly studied. The first result was obtained by V. A. Fock and F. M. Kuni [1] in the one-dimensional case. A generalization of the theorem of Fock-Kuni was obtained in a series of papers for holomorphic functions in several variables [2]. The problem of continuation of functions given on a part of the boundary of a plane domain to this domain as a solution of a generalized Cauchy-Riemann system, i.e., as a generalized analytic function has been considered in [3]. The natural analog of the Cauchy-Riemann system in three-dimensional space is the Moisil-Theodoresco (MT) system. In this talk we consider the problem of describing the functions given on a part of the boundary of a three-dimensional domain which can be continued to this domain as a solution of the (MT) system. Our investigation is based on the analog of the Cauchy integral formula for the (MT) system and a jump formula for limiting values of the Cauchy-type integral [4]. Continuation for the solution of the (MT) system to the domain by its values on a part of the boundary is based on constructing the Carleman matrix for the (MT) system. The notion of Carleman function was introduced by M. M. Lavrentev [5]. The Carleman matrix for the (MT) system was constructed in [6]. By using the continuation formula we found necessary and sufficient conditions for the extendibility of functions given on a part of a boundary to the domain as a solution of the (MT) system. We prove the Fock-Kuni theorem for the (MT) system.

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Real and complex aspects of sub-Riemannian geometry for hypoelliptic operators

The study of second order hypoelliptic operators leads naturally to sub-Riemannian geometry. The associated geodesics correspond to critical points of a complex analytic action associated by the symbol.

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Rational solutions of the Schlesinger system

Rational solutions of the Schlesinger system will be described.

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The most visited region of the Lotka-Volterra system

Lotka-Volterra system is a nonlinear system of ordinary differential equations used to model predator-prey coexistence. Rothe (1985) has derived asymptotics of the times the system spends above and below its equilibrium as its period increases ("Thermodynamics, real and complex periods of the Volterra model", Z. Agnew Math. Phys. 36, 395-421). We show that a vast majority of time one of the populations is exponentially small.

This is joint work with R. Khasminskii.

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Loewner chains in several complex variables

In this talk we discuss classical and recent results concerning the theory of Loewner chains in several complex variables.

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The Schwarzian derivative and complex Finsler metrics

In the talk, we discuss the extent to which a conformal map f of the disk onto a Jordan domain $D \subset \widehat{\mathbb{C}}$ controls the geometric and conformal invariants of the complementary domain $D^* = \widehat{\mathbb{C}} \setminus \overline{D}$. By controlling, we mean the sharp quantitative distortion of those invariants. The answer is given in terms of the Schwarzian derivative $S_f = (f''/f')' - (f''/f')^2/2$ of the Riemann mapping function f and involves its Grunsky coefficients $\alpha_{mn}(f)$.

Let $\Delta = \{|z| < 1\}$, $\Delta^* = \widehat{\mathbb{C}} \setminus \Delta$; $\mathbf{B}$ denote the complex Banach space of hyperbolically bounded holomorphic functions φ in Δ^* with $\|\varphi\| = \sup_{\Delta^*}(|z|^2 - 1)^2|\varphi(z)|$; $A_1 = A_1(\Delta)$ be the Bergman space in Δ, and let $A_1^2 \subset A_1$ denote the subset of the squares of integrable holomorphic functions in Δ. One has the pairing $\langle \mu, \psi \rangle_\Delta = \iint_D \mu\psi dz \wedge d\overline{z}$, where $\mu \in L_\infty, \psi \in L_1$.

Define for $f(z) = z + \sum\limits_1^\infty b_n z^{-n} \in \Sigma$ its *Grunsky constant*

$$\varkappa(f) = \sup \Big| \sum_{m,n=1}^{\infty} \sqrt{mn}\ \alpha_{mn} x_m x_n \Big|$$

taking the supremum over $\mathbf{x} = (x_n) \in l^2$ with $\|\mathbf{x}\| = 1$. Our key result is:

Theorem 1. *Suppose that $\|\varphi\|_{\mathbf{B}} < 2$ (consequently, the solutions of the Schwarzian equation $S_f = \varphi$ are conformal maps $f:\ \Delta^* \to \widehat{\mathbb{C}}$ having quasiconformal extensions across the unit circle S^1). Then the following properties are equivalent:*

(i) the Schwarzian derivative $\varphi = S_f$ *satisfies*

$$\sup_{\|\psi\|_{A_1(\Delta)}=1} |\langle \nu_\varphi, \psi \rangle_\Delta| = \sup_{\|\psi\|_{A_1^2(\Delta)}=1} |\langle \nu_\varphi, \psi \rangle_\Delta|,$$

where $\nu_\varphi(z) = \frac{1}{2}(1-|z|^2)^2\varphi(1/\overline{z})1/\overline{z}^4$ *is the corresponding harmonic Beltrami coefficient of the Ahlfors-Weill extension of* f *on* Δ*;*

(ii) the extremal (minimal) dilatation $k_0(f) = \inf\{k(w^\mu) = \|\mu\|_\infty : w^\mu|\overline{\Delta}^* = f\}$ *of quasiconformal extensions of* f *equals its Grunsky constant:* $k_0(f) = \varkappa(f)$.

The proof of this theorem relies on the complex metric geometry of the universal Teichmüller space $\mathbf{T}$ and on some results about the holomorphic curvatures of complex Finsler metrics. Another main tool is provided by the Grunsky inequalities.

Theorem 1 allows us to solve some problems about the quantitative estimation of the Fredholm eigenvalues and of the reflection coefficients of quasicircles as well as on extremal quasiconformal extensions, their uniqueness, etc.

The exact bound for an admissible range of the Schwarzians φ in Theorem 1 arises explicitly in the proof and is determined by the topological structure of the connected component Ω_φ^0 of the intersection of the complex line $\{t\varphi : t \in \mathbb{C}\} \subset \mathbf{B}$ with the universal Teichmüller space $\mathbf{T}$; this component contains the origin. This approach works well also for more general complex curves in $\mathbf{T}$.

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Jacobian of the complex moments mapping: general case

The theme of the talk is the algebraic properties of the complex moments mapping which arises in the potential analysis of simply-connected domains. Namely, let D be a domain in the complex plane and n is an integer. Then the complex moment of D of degree n is the quantity

$$M_n(D) = \frac{1}{\pi}\int_D (x+iy)^n dxdy.$$

This sequence is well-known in the potential analysis (in particular, it is a discrete form of the Cauchy transform of D). On the other hand, it concerns the Hele-Shaw problem of the initial liquid cell with the polynomial data when D is a quadrature domain of a kind $D = P(U)$ where P is a polynomial and U is the unit disk. It is well-known that in the last case, the sequence $\{M_k(P)\}$ is finite and gives a mapping of the coefficients body into $\mathbb{C}^n$, $n = \deg P$. We discuss the explicit formulae for the Jacobian of this mapping in the case when $D = P(U)$ and extend in various directions the conjectured Ullemar formula for the Jacobian. Our main result provides the expression for $\det J$ in terms of the roots of the derivative $P'(z)$. Acknowledgement: This work is supported by the G. Gustafsson foundation.

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Removable singularities of CR functions on singular hypersurfaces

The problem of analytic representation of integrable CR functions on hypersurfaces with singularities is treated. The nature of singularities does not matter while the set of singularities has surface measure zero. For simple singularities like cuspidal points, edges, corners, etc., the behaviour of representing analytic functions near singular points is studied.

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On univalent functions starlike with respect to a boundary point

The object of the talk is to present some recent results concerning the class of univalent functions starlike with respect to a boundary point. A method based on Julia's lemma and on properties of a dynamical system built for the class considered is developed.

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A straightening theorem for a class of meromorphic functions

The Straightening Theorem of Douady and Hubbard states that polynomial-like maps are quasi-conformally conjugate to polynomials. We prove a similar result for maps with a singular point, which are limits of polynomial-like maps. Corresponding conjugate maps are transcendental meromorphic functions. Joint work with Greg Swiatek.

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Hausdorff operators on the real Hardy space

Each integrable function generates a Hausdorff operator. It turns out that such operators are bounded taking the real Hardy space into itself. Various properties of these operators are studied, first of all those related to the Hilbert transform and to the Fourier transform.

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Optimal estimate for extrapolation from a finite set in the Wiener class

The report is devoted to the problem of analytic extension outside of a finite set from inaccurate data in a Hilbert space $H(D)$ of analytic functions in a domain $D \subset \mathbb{C}^n$.

Let $U = \{z_1, ..., z_N\}$ be a subset of distinct points in a domain D and let $z_0 \in D \setminus U$ be a fixed point. We put $V = \{f \in H(D) : \ ||f|| \le r\}$, where $r > 0$. Suppose $f_j = f(z_j) + \zeta_j, \quad j = 1, ..., N$ are approximate values of function $f \in V$ such that the errors vector $\zeta = (\zeta_1, \zeta_2, ..., \zeta_N)$ satisfies the assumption: $\zeta \in S_\delta = \{\zeta \in \ \mathbb{C}^N : \ |\zeta| \le \delta\}$ for every $f \in V$. We denote $|\zeta|^2 = \sum_{j=1}^{N} |\zeta_j|^2$, and $\delta > 0$ is a fixed number.

For characteristic of quality of extrapolation of any function $f \in V$ to $z_0 \in D \setminus U$ we consider the *optimal error of analytic extension*

$$\Omega_N(z_0;\delta) = \inf\{E_N(z_0;\alpha;\delta) : \ \alpha \in \mathbb{C}^{\mathbb{N}}\}, \tag{1}$$

where

$$E_N^2(z_0;\alpha;\delta) = \sup\{|f(z_0) - \sum_{j=1}^{N} \alpha_j f(z_j)|^2 + \left|\sum_{j=1}^{N} \alpha_j \zeta_j\right|^2 : \ f \in V, \ |\zeta| \le \delta\}. \tag{2}$$

The idea of introduction of this notion is similar to the *method of least squares* by K. Miller [1], and it differs from the estimation method by A. A. Melkman, C. A. Micchelli [2]. The main results of the talk were published in [3].

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Extremal widths on the Heisenberg group

We define the extremal length and extremal width of horizontal vector measures associated with sub-elliptic equations on the Heisenberg group. The coincidence between the module of the horizontal vector measure system and various definitions of capacity is proved. We show the continuity property of the module of a special family of curves and reciprocal relations between extremal lengths and extremal widths.

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Holomorphic Lefschetz formula for manifolds with boundary

The classical Lefschetz fixed point formula expresses the number of fixed points of a continuous map $f : M \to M$ in terms of the transformation induced by f on the cohomology of M. The purpose of the talk is to present a holomorphic Lefschetz formula on a compact complex manifold with boundary.

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Representation of functions by power series

I will discuss the following problem : which functions on the circle can be decomposed in a series of exponentials with positive frequencies?

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Radon transforms on matrix spaces

We consider the Radon transform Rf which assigns to a function $f(x)$ on the space $M_{n,m}$ of real $n \times m$ matrices $x = (x_{i,j})$ the collection of integrals over planes in $M_{n,m}$. A connection between Rf and Gårding-Gindikin fractional integrals associated to the cone of positive definite matrices is established. By using this connection, we obtain Abel type representations and explicit inversion formulas for Rf and the corresponding dual transform. It is assumed that f belongs to $L^p(M_{n,m})$ or the space of continuous functions with minimal rate of decay at infinity. This is a joint work with Boris Rubin.

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Darboux equation and reconstruction from spherical means

A construction of the singular fundamental solution for the adjoint Darboux equation will be presented. Applications to reconstruction of a function from data of spherical means will be discussed.

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Quasinormal families of meromorphic functions omitting a holomorphic function

We study the normality and quasinormality of families of meromorphic functions on plane domains. Our main results are the following.

1. Let F be a family of functions meromorphic on the plane domain D, all of whose poles are multiple and whose zeros all have multiplicity at least 3. Let $h(z)$ (not identically equal to 0 or infinity) be a function meromorphic on D. If, for each $f \in F$, $f'(z) \neq h(z)$ for all $z \in D$, then F is a normal family on D.

2. Let F be a family of functions meromorphic on the plane domain D, all of whose zeros are multiple. Suppose that for each $f \in F$, $f'(z) \neq 1$ for all $z \in D$. Then if F is quasinormal on D, it is quasinormal of order 1 there.

This is joint work with Shahar Nevo and Larry Zalcman.

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Essential spectrum of Schrödinger operators

We consider the electromagnetic Schrödinger operator

$$H = \sum_{j=1}^{n} (i\nabla_j - b_j(x)I)^2 u(x) + a(x)I,$$

where $a(x), b_j(x) \in L^{\infty,rich}(\mathbb{R}^n)$, which is considered as unbounded operator in $L^2(\mathbb{R}^n)$. Here $L^{\infty,rich}(\mathbb{R}^n)$ is a subalgebra of functions $c \in L^\infty(\mathbb{R}^n)$ with the following property: every sequence $h_m \to \infty$ has a subsequence h_{m_k} such that there exists a strong limit

$$s - \lim_{k\to\infty} c(x + h_{m_k})I = c^h(x)I,$$

an operator multiplication by the function $c(x) \in L^\infty(\mathbb{R}^n)$. The operator H_h which obtained from H by changing of $a(x), b_j(x)$ to $a^h(x), b_j^h(x)$ is called the limit operator of H defined by the sequence h_{m_k}. We denote by $Lim(H)$ the set of all limit operators of H. The following theorem give a full description of the essential spectrum of H.

Theorem. $ess\, sp\, H = \bigcup_{H_h \in Lim(H)} sp\, H_h$.

There are numerous cases in which the limit operators have a structure simple enough to allow us to give an effective description of essential spectrum of electromagnetic Scrödinger operators for new interesting classes of potentials.

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A modification of the modified equations method

One of the standard approaches used to compare solutions of differential equations and the results of numerical simulations is the so-called "modified equations method". This consists of hunting down a different differential equation for which the numerical simulation is exact, and then comparing between the original differential equation and the modified one. We show that except of the simplest cases, there are qualitative features of the dynamics of numerical simulation that the standard modified equations method can simply not capture. We propose a modification in which the modified equations are not differential but integro-differential, and give a much more accurate picture of the true behavior of the numerics.

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Representation of isotropic harmonizable covariances

A random field on $\mathbb{R}^n$ with finite variances is called *isotropic* if its means are rotationally invariant and its covariances are invariant under the natural diagonal action of $SO(n)$ on $\mathbb{R}^n \times \mathbb{R}^n$. The theory of weakly harmonizable random fields is well established, while several authors, such as Yaglom, Yadrenko, and Swift, have studied stationary and nonstationary isotropic processes and fields. In this work, conducted jointly with M.M. Rao, we study the spectral theory of isotropic, weakly harmonizable random fields and discuss an integral representation for their covariances.

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Ellipticity on spaces with corners

The program to analyse elliptic differential (and pseudo-differential) operators on manifolds with conical points, edges, or higher corners, is connected with the control of additional symbolic levels. A special case are boundary value problems in the interpretation of manifolds with edges where the boundary is the edge and the inner normal the model cone of the corresponding local wedges. In the case of 'higher' (say, regular) singularities we have a hierarchy of principal symbols, where most of the components are operator-valued. The Fredholm property is suitable scales of weighted Sobolev spaces requires additional conditions on the strata of lower dimension. We illustrate this in the case of manifolds with corners where the base is a manifold with edges. An alternative description is the case of manifolds with edges who have conical singularities.

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Pluripolar hulls and pseudocontinuation

We discuss a generalized analytic continuation expressed in terms of the *plurisubharmonic hull* of the graph of a meromorphic function. In particular, we give an example of a function f holomorphic in the unit disc such that f does not have a *pseudocontination* across any subset E of the unit circle of positive measure, while there exists a meromorphic function F in $\{|z| > 1\}$ such that:

If U is a function plurisubharmonic on $\mathbb{C}^2$ such that $U(z, f(z)) = -\infty$ on the unit disc then $U(z, F(z)) = -\infty$ for all z with $|z| > 1,\ F(z) \neq \infty$.

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Inverses under convolution

We investigate classes of univalent functions for which there exist univalent inverses under convolution.

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Harmonic functions and sign changes

This is a report on a joint with Leonid Polterovich work in progress.

Let u be a harmonic function in the unit disc $\mathbb{D}$ continuous up to the boundary, $u(0) = 0$, and let N be a number of sign changes of u on the unit circle. Then

$$\text{Area}\{z \in \mathbb{D} : u(z) > 0\} \geq \frac{C}{N},$$

where C is a positive numerical constant. The estimate is sharp up to the choice of C.

This gives a quantitative version of a result of Nadirashvili. We shall also discuss applications to metric geometry of nodal domains of eigenfunctions of Laplacian on two-dimensional surfaces.

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Holomorphic Lefschetz formula

We show a Lefschetz fixed point formula for holomorphic functions in a bounded domain D with smooth boundary in the complex plane. To introduce the Lefschetz number for a holomorphic map of D, we make use of the Bergman kernel of this domain. The Lefschetz number is proved to be the sum of usual contributions of the fixed points of the map in D and contributions of all boundary fixed points, these latter being different for attracting and repulsing fixed points.

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The level-sets analysis of holomorphic functions

We discuss our recent progress concerning the length functions on the generalized lemniscates. Let $f(z)$ be a holomorphic function with non-empty zero set. Then a compact component $E_t(f)$ of the level set $|f(z)| = e^t$ is called the generalized lemniscate of f. The famous Erdos Conjecture asks whether the length of the polynomial lemniscate $|P(z)| = 1$ is greatest in the case when $P(z) = z^n - 1$. We obtain the following

Theorem. *Given a holomorphic function $f(z)$, the length function $|E_t(f)|$ can be represented as the bilateral Laplace transform of a positive measure μ_f*

$$|E_t(f)| = \int_{-\infty}^{+\infty} e^{xt} d\mu_f(x)$$

in every regular interval $t \in \Delta$, which does not contain singular values of f. In particular, $E_t(P)$ is an exponentially convex function.

We also obtain explicit formulae for the derivatives of the length function $|E_t(f)|$ in terms of the associated Hilbert space (which leads to the positivity of the sequence of successive derivatives and to the representation above).

Corollary. *If the polynomial P is extremal for the Erdos conjecture, it has a singular point on the lemniscate $|P| = 1$.*

This results refines the recent theorem due to Eremenko & Hayman in Mich. Math. J., 1999. We also give a detailed discussion of the structure of the family of close to extremal polynomials (the so-called D-polynomials) which have their singular values on a single lemniscate. In particular, we obtain lower estimates on the length function if the function f has a singular value with high multiplicity. Another merit of our approach is that it implies the explicit formula (in terms of Gauss' hypergeometric function) for the length function when f satisfies the equation

$$f' = (1 - f^k)^m.$$

We also discuss some aspects of potential and analytic properties of the length functions in the case of D-polynomials.

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Flows on homogeneous spaces and a parametric method for conformal maps with quasiconformal extension

We study flows on the universal Teichmueller space T and on the homogeneous Kirillov space Diff S^1/S^1 embedded in T. As a result, we construct a parametric representation for conformal maps which admit quasiconformal extension and, in particular, such that the associated quasidisks are bounded by smooth Jordan curves. The Douady-Earle extension is used. Some applications to Hele-Shaw flows of viscous fluids are given.

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Semigroups of holomorphic mappings on the unit disk with a boundary fixed point

In this talk we study the class of all holomorphic semigroup generators on the open unit disk having boundary null points. This class arises, in particular, in many models of stochastic branching Markov processes.

Further, we give an asymptotic representation of starlike functions of the so-called Robertson class. We obtain a criterion of conformality of such functions at boundary null points. This is joint work with M. Elin and D. Shoikhet.

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Geometry of Carnot-Caratheodory spaces and differentiability of mappings

We prove cc-differentiability of the Lipschitz mappings on Carnot-Carathéodory spaces. As a consequence, we obtain some results of geometric measure theory and related topics.

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On the Fatou set of holomorphic maps

We discuss a few results on topological properities of the Fatou components of certain transcendental holomorphic maps.

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The Beltrami equation and finite mean oscillation

Existence and representation theorems for ACL solutions of the Beltrami equation with the dilatation majorized by functions of finite mean oscillation will be considered. This is a joint work with Vladimir Ryazanov and Uri Srebro.

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Center-Focus Problem, Moments, and Compositions — Some New Developments

The talk will present an overview of the mutual relations between the subjects mentioned in the title, stressing some new developments:
— Center equations as a non-linear deformation of the Moment equations;
— Counterexamples to the Composition Conjecture for the one-sided Moments, as well as an "almost complete" positive answer to this problem, found recently by F. Pakovich;
— Geometry of the Center equations versus the Moment equations.

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Szegö, Killip–Simon and others

A major breakthrough has been made in the spectral theory of Jacobi matrices in the case when the spectral measure σ is supported on an interval E plus *a denumerable* set of mass points which accumulate at the boundary points of E only. Together with Franz Peherstorfer we extended the famous Szegö Theorem on asymptotic for the orthonormal polynomials on the case when in addition to the Szegö's condition on E the mass points satisfy Blaschke's condition. Sergey Denisov proved the Nevai conjecture, which extend Rakhmanov's Theorem. Killip and Simon found an exact condition on the coefficients of a Jacobi matrix such that σ' is in Quasi Szegö on E and the mass points in Quasi Blaschke (this result is to appear in Annals of Mathematics). We present a general point of view on results of this kind.

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Trace and residue calculus

I will present recent developments of multidimensional residue and trace calculus, together with applications to inverse problems related to the geometric Radon transform, to division problems in polynomial rings and to arithmetic intersection or division theory.

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***Lh*-potential theory and its applications in harmonic analysis**

Lh-potential theory, dealing with so-called *Lh*-functions and maximal *Lh*-functions, is a proper tool for studying of Hadamard-type inequalities for harmonic functions in $\mathbb{R}^n$. It serves harmonic functions much as complex potential theory does for analytic functions of several complex variables, dealing with plurisubharmonic and maximal plurisubharmonic functions.

We discuss some applications to studying of orthogonal harmonic polynomials on compact subsets in and related approximation problems for harmonic functions. In that connection, we consider also some natural Lh-analogues of Green function, capacity, transfinite diameter, Chebyshev constants for a compact set in $\mathbb{R}^n$.

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A glance backward, a long look ahead

A selective (and, necessarily, much abbreviated) account of my mathematical doings over the past forty years, with an emphasis on some of my favorite unsolved problems.

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Cesaro means and resolvents of linear operators

This joint work with Juan Sanchez relates the growth of powers and their Cesaro means to the behaviour of the partial sums of the resolvent of a Banach space operator.