

**Lev Aizenberg and Elijah Liflyand**

Bar-Ilan University, Israel

*e-mail: aizenbrg@macs.biu.ac.il and liflyand@math.biu.ac.il*

**Hausdorff operators on Hardy spaces in  $\mathbb{C}^n$**

Hausdorff operators are studied on Hardy spaces in Reinhardt domains.

**Dmitry Akhiezer**

Russian Academy of Sciences, Russia

*e-mail: akhiezer@iitp.ru*

**Stein manifolds and multiplicity free representations of compact Lie groups**

We consider a Stein manifold  $M$  acted on by a connected compact Lie group  $K$  of holomorphic transformations. The action gives rise to a linear representation of  $K$  in the holomorphic function algebra  $\mathcal{O}(M)$ . We outline the proof of the following theorem.

**Theorem.** *The representation of  $K$  in  $\mathcal{O}(M)$  is multiplicity free if and only if there exists an antiholomorphic involution  $\mu : M \rightarrow M$  preserving each  $K$ -orbit. Moreover, one can choose  $\mu$  to be equivariant with respect to the Weyl involution of  $K$ .*

The proof is based on the theory of spherical varieties of reductive algebraic groups.

**Spyros Alexakis**

Massachusetts Institute of Technology (MIT), USA

*e-mail: alexakis@math.mit.edu*

**Unique continuation for the vacuum Einstein equations**

We prove a unique continuation theorem for the vacuum Einstein equations across time-like and bifurcate null surfaces. Analytically, the tool relies on using Carleman estimates for the wave operator to derive unique continuation for space-time metrics satisfying the vacuum Einstein equations. This result has been applied (in joint work with A. D. Ionescu and S. Klainerman) to the problem of uniqueness of stationary black holes.

**Lars Andersson**

Max-Planck Institut für Gravitationsphysik, Germany & University of Miami,  
USA

*e-mail: Lars.Andersson@aei.mpg.de*

### **Hidden symmetries and the wave equation on Kerr**

The wave equation on the Kerr spacetime is interesting as a toy model for the problem of stability of the Kerr black hole. In analyzing waves on Kerr one encounters two important difficulties. The Kerr spacetime has only two Killing fields, corresponding to stationarity and axial symmetry. Further, the “photon sphere”, i.e., the region filled with rotating null geodesics, has codimension zero, which makes trapping a serious problem.

In recent work with Pieter Blue, we have been able to circumvent these difficulties and give a “physical space” approach to estimates for the wave equation, by making use of the hidden symmetry of the Kerr spacetime, discovered by Carter. By utilizing the fact that an operator related to the Carter constant commutes with the Kerr wave operator, we are able to prove energy bounds, trapping, and dispersive estimates for the wave equation on Kerr.

**Robert Beig**

University of Vienna, Austria

*e-mail: robert.beig@univie.ac.at*

### **The stationary $n$ -body problem in general relativity**

I describe recent joint works with R.Schoen and with G.Gibbons and R.Schoen, which prove the non-existence of certain asymptotically flat, stationary solutions of the Einstein equations with more than one body. The basic restriction is for example satisfied when spacetime has a reflection symmetry reversing the sign of the timelike Killing vector and fixing a closed, non-compact hypersurface in the space of Killing trajectories which is disjoint from the bodies.

**Catherine Beneteau**

University of South Florida, USA

*e-mail: cbenetea@cas.usf.edu*

**Zeros of certain kernel functions in the Fock space**

(joint work with Brent Carswell and Sherwin Kouchekian)

This talk deals with certain kernel functions in the Fock space of entire functions. In particular, if we consider zero based subspaces of the Fock space, it is very easy to see that the corresponding extremal function has infinitely many additional zeros. In this talk, I will study the structure of these additional zeros under some special circumstances and give some concrete examples.

**Cinzia Bisi**

Università di Firenze, Italy

*e-mail: bisi@math.unifi.it*

**On proper polynomial maps of  $\mathbb{C}^2$**

(joint work with Polizzi Francesco)

We investigate proper polynomial maps  $g: \mathbb{C}^2 \rightarrow \mathbb{C}^2$  of arbitrary topological degree  $d$  with particular attention to the case in which  $g$  is a Galois covering. Indeed when the proper polynomial map  $g$  is invariant by the action of a finite group  $G$ , this group is a so-called *finite complex reflection group*; in this case we provide a complete classification of proper polynomial maps in 22 equivalence classes. Our work extends widely a Lamy's Theorem on proper polynomial maps of  $\mathbb{C}^2$  with topological degree 2.

**Filippo Bracci**

Università di Roma "Tor Vergata", Italy

*e-mail: fbracci@mat.uniroma2.it*

**Evolution families and the Loewner equation**

In 1923, Loewner developed a machinery to "embed" a slit domain of the complex plane into a family of domains endowed with a certain order. The key idea was to represent such domains by means of a family (nowadays known as a *Loewner chain*) of univalent functions defined on the unit disc

and satisfying a suitable differential equation. Such a machinery was then studied and extended to other types of simply connected domains by Kufarev in 1943 and Pommerenke in 1965.

Since the original paper of Loewner, this method has shown to be extremely useful when dealing with many different problems, especially those having some character of extremality. In fact, in 1984 de Branges used (extensions of) Loewner's radial equation to solve the Bieberbach conjecture. And, more recently, Schramm, and Lawler and Werner proved the Mandelbrot conjecture using a stochastic version of the so-called chordal Loewner equation.

In this talk I will discuss of some recent works with Manuel D. Contreras and Santiago Díaz-Madrigal about a full generalization (up to hyperbolic complex manifolds) of both the radial and the chordal Loewner equation.

More precisely a family  $(\varphi_{s,t})_{0 \leq s \leq t < +\infty}$  of holomorphic self-maps of the unit disc is an *evolution family of order  $d$*  with  $d \in [1, +\infty]$  (in short, an  *$L^d$ -evolution family*) if

EF1.  $\varphi_{s,s} = id_{\mathbb{D}}$ ,

EF2.  $\varphi_{s,t} = \varphi_{u,t} \circ \varphi_{s,u}$  for all  $0 \leq s \leq u \leq t < +\infty$ ,

EF3. for all  $z \in \mathbb{D}$  and for all  $T > 0$  there exists a non-negative function  $k_{z,T} \in L^d([0, T], \mathbb{R})$  such that

$$|\varphi_{s,u}(z) - \varphi_{s,t}(z)| \leq \int_u^t k_{z,T}(\xi) d\xi$$

for all  $0 \leq s \leq u \leq t \leq T$ .

Also, we call *Herglotz vector field of order  $d \geq 1$*  a function  $G : \mathbb{D} \times [0, +\infty) \rightarrow \mathbb{C}$  which is a weak holomorphic vector field of order  $d \geq 1$  (in the sense of Carathéodory's theory) and for almost every  $t \geq 0$  has the property that  $z \mapsto G(z, t)$  is an infinitesimal generator of a semigroup of holomorphic self-maps of the unit disc. Then we prove

**Theorem 1.** *For any evolution family  $(\varphi_{s,t})$  of order  $d \geq 1$  in the unit disc there exists a (essentially) unique Herglotz vector field  $G(z, t)$  of order  $d$  such that for all  $z \in \mathbb{D}$*

$$\frac{\partial \varphi_{s,t}(z)}{\partial t} = G(\varphi_{s,t}(z), t) \quad \text{a.e. } t \in [0, +\infty). \quad (1)$$

*Conversely, for any Herglotz vector field  $G(z, t)$  of order  $d \geq 1$  in the unit disc there exists a unique evolution family  $(\varphi_{s,t})$  of order  $d$  such that (1) is satisfied.*

Such a theorem is generalized to complete hyperbolic complex manifolds.

In the unit disc case, a Berkson-Porta type formula for Herglots vector fields is also discussed.

## References

- [1] F. Bracci, M.D. Contreras, and S. Díaz-Madrigal, *Evolution Families and the Loewner Equation I: the unit disc*, Preprint 2008.
- [2] F. Bracci, M.D. Contreras, and S. Díaz-Madrigal, *Evolution Families and the Loewner Equation II: complex hyperbolic manifolds*, Math. Ann., in press.

**Melkana Brakalova**

Fordham University, USA

*e-mail: brakalova@fordham.edu*

### Circle-like behavior and asymptotic homogeneity

We study the local behavior of homeomorphisms that are ACL solutions to the relaxed (degenerate) Beltrami equation. In particular we study circle-like homeomorphisms (maps with circular dilatation equal to 1). A special case of such homeomorphisms are those homogeneous at a point (asymptotic homogeneity). Using modules of ring domains we provide sufficient and necessary conditions, for a homeomorphism to be circle-like at a point. These results are applied to obtain new geometric conditions for homogeneity at a point, that closely relate to our previous work on conformality at a point and supplement some of the results in an inspirational paper of Gutlyanskii and Ryazanov (1995) on the local behavior of quasi-conformal mappings. We provide new asymptotic estimates for the behavior of both the absolute value and the argument of the map and new analytic conditions under which these estimates hold, while bypassing the assumption of K-quasiconformality.

**Tiziano Casavecchia**

Università di Pisa, Italy

*e-mail: t.casavecchia@gmail.com*

**A rigidity condition for generators in strongly convex domains**

Let  $F$  be an infinitesimal generator of a semigroup of holomorphic self-maps of the unit disc  $\mathbb{D}$  of  $\mathbb{C}$  or of the unit ball  $\mathbb{B}$  of  $\mathbb{C}^n$ . In recent years some conditions upon  $F$  forcing  $F \equiv 0$  and hence the semigroup to be trivial, were found.

In this talk I present following result generalizing what has been done: let  $F$  be an infinitesimal generator of a semigroup of holomorphic self-maps of a smooth strongly convex subdomain  $D$  of  $\mathbb{C}^n$ ; if  $F$  vanishes in angular sense at a boundary point up to third order, that is

$$\angle \lim_{z \rightarrow \tau} \frac{F(z)}{\|z - \tau\|^3} = 0$$

for some  $\tau$  in the boundary of  $D$ , then  $F \equiv 0$  on  $D$ .

**Gui-Qiang Chen**

Northwestern University, USA

*e-mail: gqchen@math.northwestern.edu*

**Nonlinear conservation laws of mixed type in mechanics and geometry**

(joint work with M. Feldman, B. Perthame, M. Slemrod, and D. Wang)

Many nonlinear partial differential equations arising naturally in mechanics and geometry are of mixed type, including nonlinear conservation laws of mixed hyperbolic-parabolic type and mixed hyperbolic-elliptic type. The solution of some fundamental issues in these areas greatly requires a deep understanding of such nonlinear partial differential equations of mixed type. Important examples include nonlinear degenerate diffusion-convection equations and transonic flow equations in fluid mechanics and the Gauss-Codazzi system for isometric embedding in differential geometry. In this talk we will discuss some recent developments in the study of nonlinear conservation laws of mixed type through these examples with emphasis on identifying/developing unified mathematical approaches, ideas, and techniques to deal with the mixed-type problems. Further trends, perspectives, and open problems in this direction will be also addressed.

**Piotr Chruściel**

University of Oxford, UK & Université de Tours, France

*e-mail: Piotr.Chrusciel@lmpt.univ-tours.fr*

**On the characteristic Cauchy problem in general relativity**

(joint work with Yvonne Choquet-Bruhat and Jose-Maria Martin-Garcia)

I will discuss how Rendall's analysis of the characteristic Cauchy problem generalizes to higher dimensions, and adapts to light cones, using a somewhat more flexible framework.

**Justin Corvino**

Lafayette College, USA

*e-mail: corvinoj@lafayette.edu*

**Constructions of  $N$ -body solutions of the Einstein constraint equations**

(joint work with Piotr Chruściel and Jim Isenberg)

Consider a finite collection of sub-regions (bodies) inside asymptotically flat solutions of the Einstein constraint equations. We construct new asymptotically flat solutions which contain each of the specified sub-regions, glued into a common asymptotic end. The construction requires less symmetry and simpler topology than that of previous constructions.

**Matteo Dalla Riva**

Università di Padova, Italy

*e-mail: mdallari@math.unipd.it*

**A singularly perturbed nonlinear traction problem in linearized elastostatics**

We consider an application of an approach based on potential theory and functional analysis to analyze a nonlinear traction problem of linearized elasticity in a domain with a small hole. The results are obtained in collaboration with Professor Massimo Lanza de Cristoforis.

**Arthur Danielyan**

University of South Florida, USA

*e-mail: adaniely@cas.usf.edu*

**On an approximation problem of L. Zalcman**

Let  $F$  be a closed subset of the unit circle  $T$  and let  $f \in C(F)$ . The talk is devoted to the problem of uniform approximation of  $f$  on  $F$  by polynomials  $P_n$  which are uniformly bounded on the unit disk  $\Delta$ . In a particular case when  $F$  is a closed arc of  $T$ , the problem was solved by Professor Zalcman in 1982, who has also mentioned on the possibility of considering more general approximation sets instead of an arc. The present paper suggests a necessary and sufficient solution of the above problem by showing that the (simple) description of  $f$  given by Zalcman for the case of an arc in fact remains the same also in the general case. In particular the new result is an immediate extension of Zalcman's theorem.

**Michael Demuth**

Technical University Clausthal, Germany

*e-mail: mbdemuth@yahoo.de*

**Where are the eigenvalues of nonselfadjoint operators?**

Let  $A$  be a selfadjoint positive operator in a Hilbert space. We consider the discrete spectrum of non-selfadjoint operators of the form  $B = A + M$ , where  $M$  is a relatively compact operator with respect to  $A$ . The number, the moments and the distribution of the discrete eigenvalues of  $B$  are estimated. The results are applicable to Schrödinger operators with complex potential.

**Santiago Díaz-Madrigal**

Universidad de Sevilla, Spain

*e-mail: madrigal@us.es*

**Generalized Loewner chains in the unit disk**

We introduce a general version of the notion of Loewner chains which extends and unifies the classical cases of the radial and chordal variant of the Loewner differential equation as well as the theory of semigroups of analytic functions. In this very general setting, we establish a deep correspondence

between these chains and the weak solutions of some specific non-autonomous differential equations. Among other things, we show that, up to a Riemman map, such a correspondence is one-to-one. In a similar way as in the classical Loewner theory, we prove that these chains are also solutions of a certain partial differential equation which resembles (and includes as a very particular case) the classical Loewner-Kuffarev PDE. We present a number of examples in order to clarify the new aspects of the theory.

**Roman Dwilewicz**

Missouri University of Science and Technology, USA and UKSW, Poland  
*e-mail: romand@mst.edu*

**Global holomorphic approximations of Cauchy-Riemann functions**  
(joint work with Al Boggess and Dan Jupiter)

The talk will be about global holomorphic approximations of Cauchy-Riemann ( $CR$ ) functions defined on  $CR$  submanifolds in  $\mathbb{C}^n$ . Cauchy-Riemann manifolds are differentiable submanifolds in complex manifolds that satisfy some constant rank condition.  $CR$  functions, defined on  $CR$  manifolds, satisfy the so-called tangential  $CR$  equations.

**Konstantin Dyakonov**

Universitat de Barcelona, Spain  
*e-mail: dyakonov@mat.ub.es*

**Blaschke products and non-ideal ideals in Lipschitz algebras**

We study certain ideals (associated with Blaschke products) of the analytic Lipschitz algebra  $A^\alpha$ , with  $\alpha > 1$ , that fail to be “ideal spaces”. The latter means that the ideals in question are not describable by any “size condition” on the functions modulus.

**Michael Eichmair**

Massachusetts Institute of Technology (MIT), USA

*e-mail: eichmair@math.mit.edu*

**Non-variational existence problems in geometry and general relativity**

In this talk we discuss existence for a class of non-variational Plateau problems including those for apparent horizons and generalized apparent horizons in general relativity. We will discuss lower order concepts from geometric measure theory which figure prominently in our approach, and which liken horizons to minimal and constant mean curvature surfaces from a variational standpoint. We will apply these techniques to prove a recent conjecture of H. Bray's and T. Ilmanen's on the existence, the regularity, and the properties of the outermost generalized apparent horizon.

**Chiara de Fabritiis**

Università Politecnica delle Marche, Italy

*e-mail: fabritiis@dipmat.univpm.it*

**Continuous iteration in non-strictly convex domains: the polydisk case**  
(joint work with Manuel Contreras and Santiago Díaz-Madrigal)

In the case of strictly convex domains the behaviour of one parameter semigroups is well understood and the relations between fixed points and limit set are known. We will discuss which results obtained in this case can be generalized to the polydisk case. In particular we will give a characterization of the infinitesimal generators of the semigroups.

**Graziano Gentili**

Università di Firenze, Italy

*e-mail: gentili@math.unifi.it*

**A geometric theory of regular functions over quaternions**

The basic results of a new theory of regular functions of a quaternionic variable have been recently stated, following an idea of Cullen, [1]. In this talk, newly discovered features of this theory will be presented. In particular the attention will be focused on the structure of the zero-sets, the minimum modulus principle, the open mapping theorem for regular functions, and some of their consequences.

## References

- [1] G. Gentili, D. C. Struppa, A new theory of regular functions of a quaternionic variable. *Adv. Math.* **216** (2007), 279–301.

**Victor Gichev**

Omsk Branch of Sobolev Institute of Mathematics, Russia

*e-mail: gichev@ofim.oscsbras.ru*

### **Polynomial convex hulls of orbits of compact groups**

Let  $V$  be a finite-dimensional complex linear space and  $G \subset \mathrm{GL}(V)$  be a compact subgroup of  $\mathrm{GL}(V)$ . The problem of determination of polynomial convex hulls for orbits  $O_v = Gv$ ,  $v \in V$ , is solved in several cases: for the isotropy representations of bounded symmetric domains (W. Kaup and D. Zaitsev), for the compact groups in the spaces of matrices, and in some more special situations. Also, it is known that the polynomially convex orbits of  $G$  are the same as the real forms of closed  $G^{\mathbb{C}}$ -orbits (Gichev and Latypov). In the talk, we consider the following two types of orbits: the adjoint orbits of a maximal compact subgroup  $G$  of a simple complex Lie group  $G^{\mathbb{C}}$  in its Lie algebra  $\mathfrak{g}^{\mathbb{C}}$  and the orbits which are commutative as homogeneous spaces (a compact homogeneous space  $M = G/H$  is commutative if the algebra of bounded operators in  $L^2(M)$  which commute with  $G$  is abelian).

**Anatoly Golberg**

Holon Institute of Technology, Israel

*e-mail: golberga@hit.ac.il*

### **Homeomorphisms with integrally restricted moduli**

The quasi-invariance of conformal module is one of underlying and characteristic features of quasiconformality. There were many attempts to weaken and generalize this property. For example, it was introduced by Martio-Ryazanov-Srebro-Yakubov in 2004 a notion of  $Q$ -homeomorphisms disturbing the conformal moduli of curves families via

$$M(f(\Gamma)) \leq \int_G Q(x) \rho^n(x) dx,$$

where  $Q(x)$  is a prescribed measurable function in a domain  $G \subset \mathbb{R}^n$ ,  $n \geq 2$ ;  $\rho$  runs over the collection of admissible functions for a curve family  $\Gamma$ , and this inequality must hold for all curve families  $\Gamma$  in  $G$ .

We consider more general classes of homeomorphisms, satisfying

$$M_\alpha(f(\mathcal{S}_k)) \leq \int_G Q(x) \rho^\alpha(x) dx,$$

and call such maps  $(\alpha, Q)$ -homeomorphisms. Here  $M_\alpha(\mathcal{S}_k)$  denotes the  $\alpha$ -module of the family of  $k$ -dimensional surfaces in  $G$  ( $1 \leq k \leq n - 1$ ).

Our goal is to establish the important differential and geometric properties of such homeomorphisms (for example, whether  $f \in W_{loc}^{1,1}$ , differentiability almost everywhere, obeying  $(N)$ -property, distortion of  $\alpha$ -module, integrability of certain local characteristics, etc.).

### Avraham Goldstein

The City University of New York, USA

*e-mail: avi\_goldstein@netzero.net*

### Plaque inverse limit and generalized conjugations of inverse dynamical systems and their applications in holomorphic dynamics

An inverse dynamical system is a sequence

$$S = S_0 \xleftarrow{f_0} S_1 \xleftarrow{f_1} S_2 \dots$$

of Riemann Surfaces  $S_i$  and branched coverings  $f_i : S_{i+1} \rightarrow S_i$  where  $S_{i+1} = S_i$  and  $f_{i+1} = f_i$  for all  $i$ . In this work we will assume that  $1 < d = \deg(f_i) < \infty$ . We define a Plaque Inverse Limit [P.I.L.]  $S_\infty$  of  $S$  as the set of all sequences of points  $x = x_0 \in S_0, x_1 \in S_1, \dots$ , such that  $f_{i+1}(x_{i+1}) = x_i$ , equipped with the topology whose base is the set of all sequences of open sets  $U = U_0 \subset S_0, U_1 \subset S_1, \dots$  such that  $f_i(U_{i+1}) = U_i$ .

Note that the Plaque Inverse Limit differs in its topology from the [standard] Inverse Limit [with the Tychonoff topology] - P.I.L. has more open sets and, thus, the identity map of sets from the P.I.L. onto the Inverse Limit is continuous. Hence the P.I.L.  $S_\infty$  is naturally equipped with continuous projection maps  $p_i : S_\infty \rightarrow S_i$  such that  $f_i \circ p_{i+1} = p_i$ .

The local base of topology of  $S_\infty$  at a point  $x$  consists of all open sets  $x \in U$  where each  $U_i$  is conformally equivalent to the unit disk in  $\mathcal{C}$  and

$f_i$ , restricted to  $U_{i+1}$ , is conformally equivalent to  $z^j$  for some  $1 \leq j \leq d_i$ . Such open sets  $U$  are called plaques. When we speak of a neighborhood of a point in  $S_\infty$  we assume it to be open and a plaque. A point  $x \in S_\infty$  is called regular if for some neighborhood  $U$  of  $x$  exists  $n$  such that  $U_{n+i+1}$  contains no critical points of  $f_{n+i}$  for all  $i = 0, 1, \dots$  [so  $f_{n+i} : U_{n+i+1} \rightarrow U_{n+i}$  is a conformal equivalence]. Otherwise point  $x \in S_\infty$  is called irregular.

Clearly, the set  $\Delta$  of all regular points of  $S_\infty$  is open and each of its path-connected components has a Riemann Surface structure. These Riemann Surfaces were studied and fully classified by Mikhail Lyubich and Yair Minski. In this work we will show that  $\Delta$  is not empty. We will also show that a point  $x \in S_\infty$  is irregular if and only if exists a neighborhood  $U$  of  $x$  such that for any neighborhood  $V$  of  $x$ , such that  $\bar{V} \subset U$ , the open set  $V - \{x\}$  is an uncountable union of disjoint path-connected components. Thus at an irregular point  $x \in S_\infty$  the P.I.L. is not even a topological manifold.

For an inverse dynamical system  $S$  we define Thickened Plaque Inverse Limit [T.P.I.L.]  $\dot{S}_\infty$  as the set of all sequences of points  $x = x_0 \in S_0, x_1 \in S_1, \dots$ , such that each sequence  $x_n, f_{n+1}(x_{n+1}), f_{n+1} \circ f_{n+1}(x_{n+2}), \dots$  is converging to some  $y_n \in S_n$ , for all  $n = 0, 1, \dots$ , equipped with the topology whose base consists of all sequences of open sets  $U = U_0 \subset S_0, U_1 \subset S_1, \dots$  such that  $f_i(U_{i+1}) = U_i$ . Clearly,  $S_\infty$  is a subspace of  $\dot{S}_\infty$ . Mapping each point  $x = x_0, x_1, \dots$  of  $\dot{S}_\infty$  to point the  $y = y_0, y_1, \dots$  of  $S_\infty$  [ $y$  belongs to  $S_\infty$  because all  $f_n$  are continuous] induces a continuous projection map  $lim : \dot{S}_\infty \rightarrow S_\infty$ . Let  $S$  and  $T$  be two inverse dynamical systems. We will show that if a sequence  $h = h_0, h_1, \dots$  of maps of sets  $h_i : S_i \rightarrow T_i$  induces a well defined map of sets  $\dot{h}_\infty : \dot{S}_\infty \rightarrow \dot{T}_\infty$  such that exists a map of sets  $h_\infty : S_\infty \rightarrow T_\infty$  with  $lim \circ \dot{h}_\infty = h_\infty \circ lim$  then  $h_\infty$  and  $\dot{h}_\infty$  are both continuous. These results will be used to define a special class of continuous maps between P.I.L. of inverse dynamical systems.

Intuitively, the classical conjugations  $f \circ h = h \circ g$  and  $h \circ f = g \circ h$  of holomorphic maps  $f$  and  $g$  can be generalized to requiring only that  $f^n \circ h$  gets "infinitely close to"  $h \circ g^n$  and  $h \circ f^n$  gets "infinitely close to"  $g^n \circ h$  as  $n \rightarrow \infty$ . Using our results, we will state precise definitions of such generalized conjugations and explore their properties.

In this work we will also develop a  $\sigma$ -Algebraic machinery which will allow us to obtain and compute certain invariants of P.I.L. under the special class of continuous maps, mentioned above. Finally, all these results will be related to Dynamical properties of Holomorphic Dynamical Systems of the iterates of rational self-mappings of the Riemann Sphere.

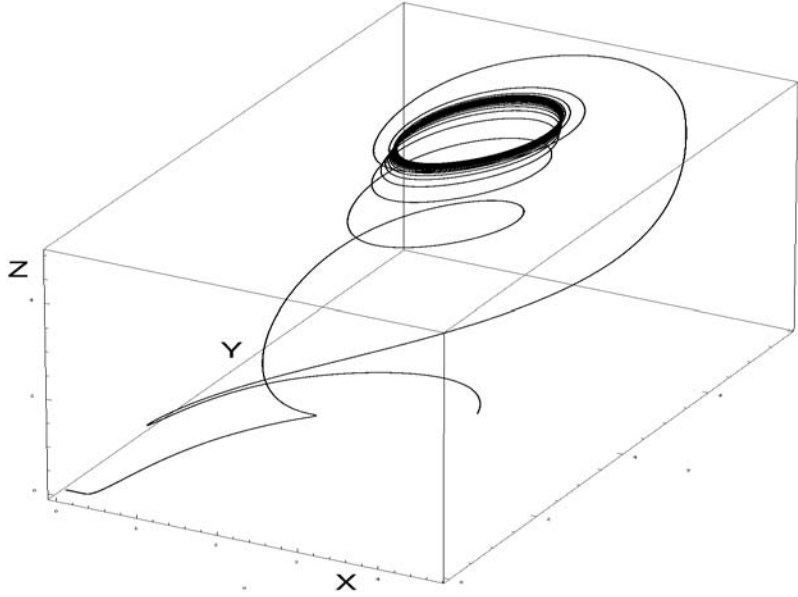


Figure 1:

**Vladimir Golubyatnikov**

Sobolev Institute of Mathematics, Russia

*e-mail: glbtn@math.nsc.ru*

**Dynamics in *p53-Mdm2* DNA damage repair network**  
(joint work with E. Mjolsness)

We study phase portrait of a system of 4 nonlinear differential equations of biochemical kinetic as a model of oscillations in the *p53-Mdm2* DNA damage repair network proposed by V.Chikarmane et al.

$$\begin{aligned} \dot{x} &= \alpha_0 z + \frac{\alpha_1 x^n}{k_1 + x^n} - \gamma_1 xy - \gamma_2 x; & \dot{y} &= \alpha_2 + \frac{\alpha_3 x^4}{k_2 + x^4} - \gamma_3 y; \\ \dot{z} &= \frac{\alpha_{1s} z (B - z)}{k_{1s} + B - z} - \frac{\alpha_{2s} z}{(k_{0d} + D)(k_{2s} + z)}; & \dot{D} &= -\alpha_d D x z. \end{aligned}$$

The variables  $x = [p53]$ ,  $y = [Mdm2]$  denote concentrations of *p53* and *Mdm2*,  $z = [AtmP]$  is the switch variable,  $D(t)$  is the amount of DNA damage.  $B = [Atm] + [AtmP]$  appears from the conservation law.

For small values of  $t$  the sum of all indices of the stationary points of the vector field  $(\dot{x}, \dot{y}, \dot{z})$  in some  $Q = [0, C_1] \times [0, C_2] \times [\varepsilon, B] \subset R_+^3(x, y, z)$  equals  $-1$ . For large  $t$  when  $D \rightarrow 0$  this sum vanishes.

For small values of  $z(0)$  and large  $D(0)$ , the trajectories of the system jump into the domain  $Q$  and oscillate there till  $D$  becomes sufficiently small. The geometric properties of the graphs of the right hand sides of the equations, are much more important for bifurcations and other dynamical characteristics of the model than is their analytic representation. The direct relation of the topological index of the repair system with the presence of the DNA damage demonstrates a new approach to gene networks studies. This approach is based on the comparison of properties (in particular complexity) of continuous models and discrete models of the same gene network.

**Samuel Grushevsky**

Princeton University, USA

*e-mail: sam@Math.Princeton.EDU*

**Meromorphic differentials with real periods, and the geometry of the moduli space of Riemann surfaces**

(joint work with Igor Krichever)

Using meromorphic differentials on Riemann surfaces with prescribed singular parts and all periods real, we define a new foliation of the moduli space of Riemann surfaces with complex leaves, and give a new proof of the Diaz' bound on the dimension of complete subvarieties of the moduli space. We will also discuss potential further application of this foliation (motivated by the Whitham perturbation theory of integrable systems) to the study of cohomology and subvarieties of the moduli space.

## **Aimo Hinkkanen**

University of Illinois at Urbana-Champaign, USA

*e-mail: aimo@illinois.edu*

### **Martingales and rotations**

We discuss a class of complex-valued functions in the plane whose complex partial derivatives can be multiplied by functions of unit modulus such that these rotated derivatives then are the end results of two martingales that are martingale transforms of each other. The motivation is that whenever this can be done, known inequalities for martingale transforms due to Burkholder then imply inequalities for various quantities associated with the partial derivatives, including their p-norms. Standard approximation results show that to get such inequalities, it suffices to consider continuous piecewise affine mappings of compact support. The problem becomes how to construct martingales by combining affine maps into new ones. We present an algorithm for doing this and describe cases where this algorithm can be carried out to the end to achieve the desired result.

## **Ilya Ioslovich**

The Technion - Israel Institute of Technology, Israel

*e-mail: agrilya@tx.technion.ac.il*

### **Optimal control of dynamical biological system: maintenance of balanced growth and development**

(joint work with Per-Olof Gutman and Raphael Linker)

We consider a dynamical biological system containing a vegetable crop and control tools for a protected and intensive cultivation system. Intensive cultivation systems - greenhouses, net houses, and plant factories- attract attention in the scope of sustainable and environmentally friendly agriculture. To optimize an economical criterion along the growing season, model based control must be designed. A special simplified biological model was developed for the purpose of determining the control inputs. This model uses the main biological properties of plant growth: the three stage process of growth (vegetative, mixed, and reproductive) resulted from the adaptation to the natural selection ontogeny, and the maintenance of the balanced sink/source ratio in relation to the growth and development processes. The appropriate optimal control problem was investigated by means of the sufficient conditions of optimality and it was found that, independently of weather inputs,

the invariant parameter optimal control intensity can be determined analytically. A previously calibrated generic comprehensive multidimensional model of the tomato plant was used as a generator of data for simulation.

**Jim Isenberg**

University of Oregon, USA

*e-mail: isenberg@uoregon.edu*

**Asymptotic gluing of asymptotically hyperbolic solutions to the  
Einstein Constraint equations**

(joint work with John M. Lee and Iva Stavrov)

We show how to glue a pair of asymptotically hyperbolic solutions of the Einstein constraint equations in such a way that their asymptotic regions are connected.

**Shulim Kaliman**

University of Miami, USA

*e-mail: kaliman@math.miami.edu*

**Algebraic density property of homogeneous spaces**

A smooth complex affine algebraic variety  $X$  has the algebraic density property (ADP) if the Lie algebra generated by completely integrable algebraic vector fields coincides with the space of all algebraic vector fields on  $X$ . If  $X$  is also equipped with an algebraic volume form (i.e. a non-vanishing section of the canonical bundle) then it has the algebraic volume density property (AVDP) when the similar fact holds for algebraic vector fields of zero divergence with respect to this form. These notions are the core of a new direction in complex analysis - the Andersen-Lempert theory. We establish ADP for a wide class of affine algebraic varieties that includes most of linear algebraic groups and homogeneous spaces, and AVDP for a class of objects that includes, in particular all semi-simple Lie groups.

**Stanislawa Kanas**

Rzeszow University of Technology, Poland

*e-mail: skanas@prz.rzeszow.pl*

To be announced

**Lev Kapitanski**

University of Miami, USA

*e-mail: levkapit@math.miami.edu*

**Analytic form of the Pontrjagin-Hopf invariants**

In this talk I will describe the homotopy invariants for maps between a three-dimensional manifold and the two-sphere. The analytical description that Dave Auckly and I have found recently extends to Sobolev maps with finite Faddeev energy. The issue of integrality of the invariants for such (discontinuous) maps is quite subtle.

**Maria Karmanova**

Siberian Branch of Russian Academy of Sciences, Russia

*e-mail: maryka84@gmail.com*

**The area formula for Lipschitz mappings of Carnot–Carathéodory spaces**

The geometry of Carnot–Carathéodory spaces naturally arises in physics, non-holonomic mechanics, optimal control theory, contact geometry, theory of subelliptic equations, tomography, neurobiology, robotechnics etc. Heisenberg and Carnot groups are particular cases of Carnot–Carathéodory spaces.

The sub-Riemannian analogs of the well-known area formula are proved in recent papers by V. Magnani [1] and S. D. Pauls [2] for Lipschitz mappings of Carnot groups  $\varphi : \mathbb{G} \rightarrow \tilde{\mathbb{G}}$ . However, in their works, these authors do not give the analytic description of sub-Riemannian Jacobian since they construct the Hausdorff measures  $\mathcal{H}^\nu$  with respect to Carnot–Carathéodory metrics. The definition is the following:

$$\mathcal{J}^{SR}(\varphi, x) = \lim_{r \rightarrow 0} \frac{\mathcal{H}^\nu(\varphi(B_{cc}(x, r)))}{\mathcal{H}^\nu(B_{cc}(x, r))} = \lim_{r \rightarrow 0} \frac{\mathcal{H}^\nu(\widehat{D}\varphi(x)[B_{cc}(x, r)])}{\mathcal{H}^\nu(B_{cc}(x, r))},$$

where the latter equality is established due to the bi-Lipschitz equivalence of the mappings  $\varphi$  and  $w \mapsto \widehat{D}\varphi(x)[w]$ . Obviously, it is impossible to calculate this value. Here  $\widehat{D}\varphi(x)$  is the  $\mathcal{P}$ -differential of  $\varphi$  at  $x$ .

We prove the sub-Riemannian area formula for Lipschitz (with respect to sub-Riemannian metric) mappings of Carnot–Carathéodory spaces. The result is new even for mappings of Carnot groups: we prove that

$$\int_A \sqrt{\det(\widehat{D}\varphi(x)^* \widehat{D}\varphi(x))} d\mathcal{H}^\nu(x) = \int_{\varphi(A)} d\mathcal{H}^\nu(y),$$

where  $\varphi \in \text{Lip}(\mathbb{M}, \widetilde{\mathbb{M}})$ ,  $A \subset \mathbb{M}$ . Here the sub-Riemannian Jacobian equals  $\sqrt{\det(\widehat{D}\varphi(x)^* \widehat{D}\varphi(x))}$  and Hausdorff measures  $\mathcal{H}^\nu$  are constructed with respect to the (sub-Riemannian) quasimetric  $d_2$  that is convenient for calculations, and  $\widehat{D}\varphi(x)$  is the  $hc$ -differential of  $\varphi$  at  $x$  [3].

Note that, we prove our result under the assumption that the basis vector fields both in the image and in the preimage are  $C^1$ -smooth. Remark that on Carnot–Carathéodory spaces, there is no bi-Lipschitz approximation of  $\varphi$  by the “tangent” mapping  $w \mapsto \widehat{D}\varphi(x)[w]$  since the quasimetrics in  $\mathbb{M}$  and in any local Carnot group are not equivalent. Moreover, the mappings that are Lipschitz with respect to sub-Riemannian metrics may not be differentiable in classical sense on sets of positive measure, thus, we cannot deduce this case to the case of Riemannian formula. So, we suggest the essentially new methods of investigation of Lipschitz mappings. The methods of proving are new even for the case of a mapping of Euclidean spaces [4].

## References

- [1] Magnani V. *Differentiability and Area formula on Stratified Lie groups* // Houston J. Math. **27** (2001). P. 297–323.
- [2] Pauls S. D. *A notion of rectifiability modeled on Carnot groups* // Indiana University Mathematics Journal **53** (2004). P. 49–82.
- [3] Vodopyanov S. *Geometry of Carnot–Carathéodory Spaces and Differentiability of Mappings* // The Interaction of Analysis and Geometry. Contemporary Mathematics, **424** (2007). P. 247–302.

- [4] Karmanova M. *The area formula for Lipschitz mappings of Carnot–Carathéodory spaces* // Dokl. AN **423** (2008).

**Victor Katsnelson**

The Weizmann Institute, Israel  
*e-mail: victorkatsnelson@gmail.com*

### **The truncated Fourier Operator**

The truncated Fourier operator  $\mathcal{F}_E$  (more precisely, the Fourier operator truncated on the set  $E$ ) is the operator of the form

$$\mathcal{F}_E = P_E \mathcal{F} P_E,$$

where  $\mathcal{F}$  is the Fourier-Plancherel operator acting in the space  $L^2(\mathbb{R})$ ,  $E$  is a subset of the real axis, and  $P_E$  is the natural orthogonal projector from  $L^2(\mathbb{R})$  onto  $L^2(E)$ .

Spectral properties of the operator  $\mathcal{F}_E : L^2(E) \rightarrow L^2(E)$  are discussed.

**Mikhail Kharlamov**

Volgograd Academy of Public Administration, Russia  
*e-mail: mharlamov@vags.ru*

### **New examples of hyperelliptic integrable Hamiltonian systems**

New integrable almost Hamiltonian systems with two degrees of freedom naturally appear as the dynamical systems induced on critical submanifolds of the integrable systems of higher dimensions. We consider the Reyman–Semenov-Tian-Shansky generalization of the Kowalevski case in the dynamics of rigid body. This problem is not reducible to a family of systems with two degrees of freedom and no solutions or separation of variables are found yet. We propose the complete description of the critical submanifolds and, consequently, of the bifurcation diagram of the general problem. For two critical subsystems the separation of variables is found giving the systems of Kowalevski type but with different degrees of polynomials. Thus we obtain the examples of the systems having hyperelliptic solutions. It is interesting to note that these systems describe all the components of the rigid body motion while in the classical problems we determine the motions only up to

rotations around the symmetry axis. Some classifications of the solutions and the description of the phase topology are presented for the subsystems discussed.

**Irina Kharlamova**

Volgograd Academy of Public Administration, Russia

*e-mail: irinah@vags.ru*

**Geometrical analysis of some problems of integrable tops**

In the problem of motion of a rigid body about a fixed point in the potential field of generic type, one can introduce the new complex coordinate system generalizing the first coordinate change of S. Kowalevski. The appropriate form of the geometric integrals leads to the equation of an ellipsoid involving the modules of the new complex variables. Some natural system of local coordinates on this ellipsoid is proposed. It appears of great importance in the problem of finding the analytical solutions in the integrable but not reducible problem of the Kowalevski top in double force field. Some applications to the geometrical investigation of the problem are demonstrated.

**Dmitry Khavinson**

University of South Florida, USA

*e-mail: dkhavins@cas.usf.edu*

**“Fingerprints” of the two dimensional shapes and lemniscates**

(joint work with P. Ebenfelt and H. S. Shapiro)

The newly emerging field of vision and pattern recognition often focuses on the study of two dimensional “shapes”, i.e. simple, closed smooth curves. A common approach to describing shapes consists in defining a “natural” embedding of the space of curves into a metric space and studying the mathematical structure of the latter. Another idea that has been pioneered by Kirillov and developed recently among others by Mumford and Sharon consists of representing each shape by its “fingerprint”, a diffeomorphism of the unit circle. Kirillov’s theorem states that the correspondence between shapes and fingerprints is a bijection modulo conformal automorphisms of the disk. In this talk we discuss the recent joint work with P. Ebenfelt and Harold S. Shapiro outlining an alternative interpretation of the problem of shapes

and Kirillov's theorem based on finding a set of natural and simple fingerprints that is dense in the space of all diffeomorphisms of the unit circle. This approach is inspired by the celebrated theorem of Hilbert regarding approximation of smooth curves by lemniscates. We shall outline proofs of the main results and discuss some interesting function-theoretic ramifications and open questions regarding possibilities of numerical applications of this idea.

**Sergiu Klainerman**

Princeton University, USA

*e-mail: seri@math.princeton.edu*

**Uniqueness of stationary black holes without analyticity**

I will describe recent results with Alexakis and Ionescu which remove the analyticity assumption in Hawking's famous rigidity theorem.

**Dorota Klim**

University of Łódź, Poland

*e-mail: klimdr@math.uni.lodz.pl*

**Dynamical systems in cone metric spaces**

To be announced

**Yuri Kondratiev**

Bielefeld University, Germany

*e-mail: yukondrat@yandex.ru*

**Dynamics of generating functionals for interacting particle systems**

We will discuss an approach to Markov evolutions of interacting particle systems in continuum based on the generating functionals description. In this approach the time evolution on the initial state may be reduced to a dynamical system acting in a space of holomorphic functionals on a functional space. This point of view gives not only an alternative method for the construction of the Markov evolutions, but also a rigorous possibility to derive Vlasov type kinetic equations for stochastic dynamics. We will apply corresponding scaling limit analysis to several particular models coming from mathematical physics and spatial ecology.

**Jurij Kozicki**

Maria Curie-Skłodowska University, Poland & Universität Bielefeld, Germany

*e-mail: jkozi@hektor.umcs.lublin.pl*

**Quantum stabilization of systems of interacting anharmonic oscillators**

We show that the dynamics of a system of interacting quantum oscillators with unstable equilibrium positions (anharmonic oscillators) can be stable, inspite of the fact that the corresponding classical dynamics is unstable. This effect is described by a parameter, which we call quantum rigidity. The parameter and the effect itself are naturally connected with the spectral properties of the corresponding Schroedinger operators.

**Samuel Krushkal**

Bar-Ilan University, Israel

*e-mail: krushkal@macs.biu.ac.il*

**Zalcman's conjecture and related problems**

In the end of 1960's, Lawrence Zalcman conjectured that the coefficients of univalent functions  $f(z) = z + \sum_2^{\infty} a_n z^n \in S$  on the unit disk satisfy the sharp inequality  $|a_n^2 - a_{2n-1}| \leq (n-1)^2$ , with equality only for the Koebe function. This remarkable conjecture implies the Bieberbach conjecture; it has been investigated by many mathematicians, and still remains a very difficult open problem for  $n > 3$ . It was proved only in certain special cases.

We provide a proof of Zalcman's conjecture based on results concerning the plurisubharmonic functionals and metrics on the universal Teichmüller space. This method gives also other new sharp estimates for large coefficients. As a corollary, this implies a new proof of the Bieberbach conjecture.

**Tadeusz Kuczumow**

University of Maria Curie-Skłodowska, Poland

*e-mail: tadek@hektor.umcs.lublin.pl*

**Intersections of holomorphic retracts**

(joint work with M. Budzynska and W. Kaczor)

We consider intersections of holomorphic retracts in bounded domains which are linearly strictly convex in the Kobayashi distance.

**Galina Kurina**

Voronezh State Forestry Academy, Russia

*e-mail: kurina@math.vsu.ru*

**Asymptotic solution of optimal control problems with intermediate points and small parameter in performance index**

(joint work with E. V. Smirnova)

The asymptotic expansion of the solution of optimal control problems without constraints for the control with fixed intermediate points and a small parameter in the performance index has been constructed as series of non-negative integer powers of the small parameter. The degenerate problems, when a small parameter is equal to zero, are optimal control problems without intermediate points. In general case, an optimal control for considered perturbed problems is a discontinuous function although it is a continuous function for corresponding degenerate problems. The estimates have been obtained for the proximity of the asymptotic approximate solutions to the exact one. The nice property is proved, namely, the values of the minimized functional do not increase when higher-order approximations to the optimal control are used. For linear-quadratic optimal control problems, the asymptotic expansion of the solution in a feedback form has been constructed using the asymptotic expansion of discontinuous solutions for the corresponding matrix Riccati equation and an auxiliary linear differential equation. The asymptotic analysis is also presented for optimal control problems the state equation of which is two sequentially acting systems. The matching conditions for the trajectories in the switch point are absent but the minimized functional depends on the values of the state trajectory in the left and right sides from the switch points. For this problem, the state trajectories are discontinuous functions. The numerical examples are given in order to illustrate the proposed method.

**Alexandr Kytmanov**

Siberian Federal University, Russia

*e-mail: kytmanov@lan.krasu.ru*

**On the asymptotic expansion of the conormal symbol of the singular Bochner-Martinelli integral**

We study the conormal symbol of the singular Bochner-Martinelli integral on a compact closed surface with conical points  $S$  in  $C^n$  and its asymptotic expansion.

**Massimo Lanza de Cristoforis**

Università di Padova, Italy

*e-mail: mlde@math.unipd.it*

**Singular perturbation problems in potential theory: a functional analytic approach**

This talk is dedicated to the analysis of boundary value problems on singularly perturbed domains by an approach which is alternative to those of asymptotic analysis and of homogenization theory.

In particular, we will consider a certain linear or nonlinear boundary value problem on a domain with one or possibly infinitely many holes, whose size is determined by a positive parameter  $\epsilon$  and we will consider a family of solutions depending on  $\epsilon$  as  $\epsilon$  approaches 0. Then we shall represent the dependence on  $\epsilon$  of the family of solutions, or of corresponding functionals of the solutions such as the energy integral, in terms of possibly singular at 0 but known functions of  $\epsilon$  such as  $\epsilon^{-1}$  or  $\log \epsilon$ , and in terms of possibly unknown real analytic operators.

**Vitali Liskevich**

Swansea University, UK

*e-mail: V.A.Liskevich@swansea.ac.uk*

**Some qualitative properties of second-order elliptic and parabolic equations**

We discuss local boundedness, continuity and removability of isolated singularities for wide range of second-order quasi-linear elliptic and parabolic equations with lower order terms from Kato type classes.

**Doron Lubinsky**

Georgia Institute of Technology, USA

*e-mail: lubinsky@math.gatech.edu*

**Universality limits for random matrices and de Branges spaces**

Universality limits for random matrices describe the spacings between successive eigenvalues of random matrices, and their distribution. One way to establish such universality limits is via the theory of entire functions of

exponential type, and de Branges spaces. We shall discuss the method and show some new results. No background on de Branges spaces, or universality is assumed.

**Mikhail Lyubich**

Stony Brook University, USA  
*e-mail: mlyubich@math.sunysb.edu*

**Renormalization and infinite-dimensional complex geometry**

Renormalization is an efficient tool in investigation of the small scale structure of fractals, like Julia sets and the Mandelbrot set. It is an operator acting in an appropriate infinite-dimensional complex space. The main problem is to prove hyperbolicity of this action. It involves introducing an appropriate hyperbolic metric in the space to which one can apply a version of the Schwarz Lemma. In the talk we will describe these structures and their applications. Based on a recent joint work with Artur Avila.

**Olimjan Makhmudov**

Samarkand State University, Uzbekistan  
*e-mail: makhmudovo@rambler.ru*

**The Cauchy problem for the system equation lame**

In this paper, we considered the problem of analytical continuation of the solution of the system equations of the moment theory of elasticity in spacious unbounded domain from its values and values of its strains on part of the boundary of this domain, i.e., the Cauchy's problem.

**Irina Markina**

University of Bergen, Norway  
*e-mail: ima083@math.uib.no*

**The notion of the sub-Lorentzian geometry**

Sub-Riemannian manifolds and the geometry introduced by bracket generating distributions of smoothly varying  $k$ -planes is widely studied, interesting subject, which has applications in control theory, quantum physics, C-R

geometry, and other areas. The main difference of the sub-Riemannian manifold from a Riemannian one is the presence of a smooth subbundle of the tangent bundle, generating the entire tangent bundle by means of the commutators of vector fields. The subbundle, which is often called horizontal, is equipped with a positively definite metric that leads to the triplet: manifold, horizontal subbundle, and Riemannian metric on the horizontal subbundle, which is called a *sub-Riemannian manifold*. The foundation of the sub-Riemannian geometry can be found in [4, 5]. The following question can be asked. What kind of geometrical features will have the mentioned triplet if we change the positively definite metric to an indefinite non-degenerate metric. It is natural to start with the Lorentzian metric of index 1. In this case the triplet: manifold, horizontal subbundle, and Lorentzian metric on the horizontal subbundle can be called *sub-Lorentzian manifold*. It was mentioned in [5] that it would be interesting to consider the sub-Lorentzian geometry, but there are only few works devoted to this subject. In the talk we present the different examples of the manifolds with the non-degenerated indefinite metric defined on the subbundle. We will compare the sub-Riemannian geometry and the sub-Lorentzian. We find the parametric equations for timelike future directed geodesics. We also present some physical applications.

## References

- [1] Chang D. C., Markina I., Vasiliev A. *Sub-Lorentzian geometry on anti-de Sitter space*, J. Math. Pures Appl. **90** (2008), no. 1, 82–110.
- [2] Grochowski M. *Reachable sets for the Heisenberg sub-Lorentzian structure on  $\mathbb{R}^3$ . An estimate for the distance function*. J. Dyn. Control Syst. 12 (2006), no. 2, 145–160.
- [3] Korolko A., Markina I. *Nonholonomic Lorentzian geometry on some  $\mathbb{H}$ -type groups*. ArXiv:0809.4450
- [4] Montgomery R. *A tour of subriemannian geometries, their geodesics and applications*. Mathematical Surveys and Monographs, **91**. American Mathematical Society, Providence, RI, 2002. 259 pp.
- [5] Strichartz R.S. *Sub-Riemannian geometry*, J. Differential Geom. **24** (1986) 221–263; Correction, *ibid.* **30** (1989) 595–596.

**David Maxwell**

University of Alaska Fairbanks, USA

*e-mail: david.maxwell@uaf.edu*

**On solutions of the Einstein constraint equations obtained by the conformal method**

In this talk we present a sufficient condition under which the conformal method can be used to generate solutions of the vacuum Einstein constraint equations on compact manifolds. The existence theorem includes as easy special cases known constant-mean-curvature (CMC) and near-CMC existence theorems, and it extends to vacuum data the recent far-from-CMC results of Holst, Nagy, and Tsogtgerel. We discuss how our understanding of the landscape of solutions obtained by the conformal method has now changed, and we indicate how the conformally flat torus might be used to discern how the current gaps in this understanding might be filled.

**Vladimir Maz'ya**

University of Liverpool and Linköping University

*e-mail: vlmaz@mai.liu.se*

**Higher order elliptic problems in non-smooth domains**

(joint work with S. Mayboroda)

We discuss sharp regularity results for solutions of the polyharmonic equation in an arbitrary open set. Then we introduce an appropriate notion of capacity which allows us to describe the precise correlation between the smoothness of the solution and the geometry of the domain.

**Michael Monastyrsky**

Institute of Theoretical and Experimental Physics, Moscow, Russia

*e-mail: monastyrsky@itep.ru*

**Hecke surfaces and duality transformations in lattice spin models**

I discuss two topics.

1. Hecke surfaces and  $k$ -regular graphs.

Hecke surfaces are the class of Riemann surfaces connected with Hecke groups and they are a natural extension of modular group. I study the

spectral characteristics of these surfaces. One of the main results is the exhibit of some universal properties of Hecke surfaces with large cusps.

2. Duality transformations for non-abelian spin systems.

Duality transformation for non-abelian lattice models is a generalization of classical duality discovered by H. Kramers and G. Wannier in Ising model. I consider more general models including Potts model and some others. Deep internal relations of all these problems have become evident during recent years. I give some examples of recent results in these fields.

**Simona Myslivets**

Siberian Federal University, Russia

*e-mail: simona@lan.krasu.ru*

**On the conditions  $\bar{\partial}$ -closed extension of differential forms**

The purpose of the talk to receive a condition  $\bar{\partial}$ -closed extension of differential forms from boundary of the bounded domain  $D \subset \mathbb{C}^n$  in terms of harmonic extension of its coefficients.

**Ikbol Niyozov**

Samarkand State University, Uzbekistan

*e-mail: iqboln@mail.ru*

**The Cauchy problem for the system of the moment theory elasticity in  $E^m$**

In this paper, we considered the problem of analytical continuation of the solution of the system equations of the moment theory of elasticity in spacious bounded domain from its values and values of its strains on part of the boundary of this domain, i.e., the Cauchy's problem.

**Anthony G. O’Farrell**

NUI Maynooth, Co. Kildare, Ireland

*e-mail: anthonyg.ofarrell@gmail.com*

**Reversible biholomorphic germs**

(joint work with Patrick Ahern)

A reversible map is a special kind of invertible map: it is one that is conjugate to its own inverse, so that the associated dynamical system behaves in the same way when run forwards or backwards in time. We describe this situation by saying that “the past is the future of an alternative present”.

Apart from the obvious examples in classical mechanics, one finds that reversible maps arise in diverse areas such as polynomial convexity, real and complex polynomial approximation, and other areas of functional analysis. These examples motivated our interest in reversibility.

The natural abstract context is the theory of groups. Let  $G$  be a group. We say that an element  $f \in G$  is *reversible in  $G$*  if it is conjugate to its inverse, i.e. there exists  $g \in G$  such that  $g^{-1}fg = f^{-1}$ . We denote the set of reversible elements by  $R(G)$ . For  $f \in G$ , we denote by  $R_f(G)$  the set (possibly empty) of *reversers* of  $f$ , i.e. the set of  $g \in G$  such that  $g^{-1}fg = f^{-1}$ .

For some time, we have studied reversibility in specific groups of mappings connected to real and complex analysis and geometry, including Möbius groups, groups of formal power series, groups of biholomorphic maps, homeomorphism groups and diffeomorphism groups. There are many open questions about reversibility, and about the related subjects of involutions and products of involutions.

In this talk, we characterise the elements of  $R(G)$  and describe each  $R_f(G)$ , where  $G$  is the the group of biholomorphic germs in one complex variable. That is, we determine all solutions to the equation  $f \circ g \circ f = g$ , in which  $f$  and  $g$  are holomorphic functions on some neighbourhood of the origin, with  $f(0) = g(0) = 0$  and  $f'(0) \neq 0 \neq g'(0)$ .

**Victor Palamodov**

Tel Aviv University, Israel

*e-mail: palamodo@post.tau.ac.il*

**Compulsory extension of solutions of analytic PDE**

Holomorphic functions of several variables submit rules of obligatory extension according to the classical theorems of Hartogs, Osgood and Oka.

This phenomenon was studied in sixties for solutions of PD systems with constant coefficients. I discuss new results describing similar properties for solutions of general overdetermined PDE with analytic coefficients.

**Boris Paneah**

The Technion - Israel Institute of Technology, Israel

*e-mail: peter@techunix.technion.ac.il*

**Noncommutative dynamical systems with two generators and their applications in analysis**

In this talk some new dynamical systems which are determined by a semi-group  $\Phi$  of maps in a closed interval  $I$  are studied. The main peculiarity of these systems is that  $\Phi$  is generated by two noncommuting maps. Introducing certain closed subsets  $\mathcal{T}_1$  and  $\mathcal{T}_2$  in  $I$  makes it possible to determine some specific orbits corresponding to  $\Phi$  and some specific attractors in  $I$ . These orbits play a crucial role in solving a wide variety problems in such diverse fields of analysis as functional and functional-integral equations, integral geometry, boundary problems for hyperbolic partial differential equations of higher ( $> 2$ ) order. In the first part of this talk we describe some conditions which ensure the existence of attractors in question of a special structure. In the second part several new problems in the above-mentioned fields of analysis are formulated, and we trace how the above dynamic approach works in solving these problems.

**Yehuda Pinchover**

The Technion - Israel Institute of Technology, Israel

*e-mail: pincho@techunix.technion.ac.il*

**On Liouville theorems for p-Laplacian-type equations**

In this talk we shall study Liouville theorems for p-Laplacian-type equations. In particular, we shall present a Liouville comparison principle for such equations. In the second part of the talk, we shall discuss positive solutions of Fuchsian p-Laplacian-type equations.

**Sergiy Plaksa**

Institute of Mathematics of the National Academy of Sciences of Ukraine,  
Ukraine

*e-mail: plaksa@imath.kiev.ua*

**Commutative algebras associated with classic equations of  
mathematical physics**

Analytic function methods in the complex plane for plane potential fields inspire searching analogous effective methods for solving spatial problems of mathematical physics. Such methods may be based on mappings of hypercomplex algebras. Ideas of an algebraic-analytic approach to equations of mathematical physics means a finding of commutative Banach algebras such that monogenic functions defined on them form an algebra and have components satisfying to beforehand given equations with partial derivatives.

A monogenic function theory in commutative Banach algebras associated with classic equations of mathematical physics is developed at the Institute of Mathematics of the National Academy of Sciences of Ukraine. Such algebras are constructed for the two-dimensional biharmonic equation and the three-dimensional Laplace equation and some other equations. Studying monogenic functions defined in mentioned algebras discovers a way to develop effective analytic methods for solving various problems of mathematical physics.

**Daniel Pollack**

University of Washington, USA

*e-mail: pollack@math.washington.edu*

**Initial Data for Vacuum Spacetimes with a Positive Cosmological  
Constant**

The time-symmetric vacuum constraint equations for the Einstein field equations are precisely the condition that a Riemannian metric has constant scalar curvature, with the sign of the scalar curvature corresponding to the sign of the cosmological constant. In the positive case, we will show how this leads to the recognition that known results for the singular Yamabe problem may be reinterpreted as results on the existence of spacetimes with asymptotically Kottler-Schwarzschild-de Sitter ends. Joint work with Piotr Chrusciel and Frank Pacard shows how one can go further to construct spacetimes

with exactly Kottler-Schwarzschild-de Sitter ends. This will be compared with analogous results for zero and negative choices of the cosmological constant.

**Dmitri Prokhorov**

Saratov State University, Russia  
*e-mail: ProkhorovDV@info.sgu.ru*

**Asymptotic coefficient estimates for regular and singular Löwner trajectories**

The Löwner differential equation serves a source for obtaining functional estimates in classes of univalent functions, coefficient estimates among them. Asymptotic results appear, e.g. in the classes of bounded univalent functions  $|f(z)| < M$  as  $M \rightarrow \infty$  or  $M \rightarrow 1$ . Another type of Taylor expansions is considered in the case when functions  $f$  map the unit disk or the upper half-plane onto these domains slit along an analytic curve. The extended inverse function  $f^{-1}$  is expanded around the tip, and the asymptotic coefficient behavior is described in terms of the time parameter  $t$  as  $t \rightarrow 0$ . Singular solutions to the Löwner equation give a chance for asymptotic estimating, and connect singularities of the driving function at  $t = 0$  in the Löwner equation with the slit parametric representation.

**Vladimir Rabinovich**

National Polytechnic Institute of Mexico, Mexico  
*e-mail: vladimir\_rabinovich@hotmail.com*

**Essential spectra and exponential estimates of eigenfunctions of lattice operators of quantum mechanics**

The main aim of the talk is the estimates of the essential spectra and exponential decreasing of eigenfunctions of difference operators on the lattice  $Z^n$ . Our investigation are based on the calculus of the so-called pseudodifference operators (pseudodifferential operators on the group  $Z^n$ ) with analytic symbols, and the limit operators method. As applications we give a description of the essential spectra and the estimates of the eigenfunctions of main lattice operators of the Quantum Mechanics:

1. Matrix Schrödinger operators on  $Z^n$ ,
2. Dirac operators on  $Z^3$ ,
3. Square root Klein -Gordon operators on  $Z^n$ .

**Martin Reiris**

Massachusetts Institute of Technology, USA

*e-mail: reiris@math.mit.edu*

**Scalar curvature, the isoperimetric inequality and the Einstein flow in the Constant Mean Curvature gauge**

We will discuss precise conjectures to understand isoperimetric collapse under integral bounds on the Ricci curvature for three-dimensional Riemannian manifolds with scalar curvature bounded below. We will then go to explain potential applications to the Einstein flow in the CMC gauge. More notably we will apply them to obtain curvature-like continuity criteria and to the extraction of complete limits in the blow up of flow singularities. Finally we will sketch a series of ideas on how to prove the conjecture using stable minimal surfaces.

**Michael Reissig**

Technical University Bergakademie Freiberg, Germany

*e-mail: reissig@math.tu-freiberg.de*

**Decay rates for wave models with structural damping**

(joint work with Xiaojun Lu (Hangzhou))

In this talk, we will present results on the behavior of higher order energies of solutions to the following Cauchy problem for a wave model with structural damping:

$$u_{tt} - \Delta u + b(t)(-\Delta)^\sigma u_t = 0, \quad u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), \\ \sigma \in (0, 1], \quad b(t) = \mu(1 + t)^\delta, \quad \mu > 0, \quad \delta \in [-1, 1].$$

We are interested in the influence of the structural dissipation (between external and visco-elastic damping)  $b(t)(-\Delta)^\sigma u_t$  on  $L^2 - L^2$  estimates.

*Our main goal is to study under which conditions do we have a parabolic effect for the solutions, that is, the decay rates depend on the order of energy.*

In the talk we will explain how *hyperbolic* or *elliptic* WKB analysis comes in. The main tools are a correct division of the extended phase space into zones, diagonalization procedures, construction of fundamental solutions and a gluing procedure. Some open problems complete the talk.

Lu/Reissig, *Rates of decay for structural damped models with decreasing in time coefficients*, 30 A4, submitted.

Lu/Reissig, *Rates of decay for structural damped models strictly increasing in time coefficients*, in preparation.

**Leiba Rodman**

College of William and Mary, USA

*e-mail: lxrodm@math.wm.edu*

**Perturbation analysis of Lagrangian invariant subspaces of symplectic matrices**

(joint work with C. Mehl, V. Mehrmann, and A. C. M. Ran)

A subspace of a finite dimensional real or complex vector space is said to be Lagrangian (relative to a given nondegenerate bilinear or sesquilinear form) if it is neutral with respect to the form and the dimension of the subspace is equal to half the dimension of the vector space. Lagrangian invariant subspaces for symplectic matrices play an important role in the numerical solution of discrete time, robust and optimal control problems. The sensitivity (perturbation) analysis of these subspaces, however, is a difficult problem, in particular, when the eigenvalues are on or close to some critical regions in the complex plane, such as the unit circle.

A detailed perturbation analysis is presented for several different cases of real and complex symplectic matrices. Various stability (robustness) properties of Lagrangian invariant subspaces of these matrices are analyzed.

**Igor Rodnianski**

Princeton University, USA

*e-mail: irod@Math.Princeton.EDU*

**Black holes and linear waves**

To be announced

**Boris Rubin**

Louisiana State University, USA

*e-mail: borisr@math.lsu.edu*

**Comparison of volumes of convex bodies in real, complex, and quaternionic spaces**

The classical Busemann-Petty problem (1956) asks, whether origin - symmetric convex bodies in  $R^n$  with smaller hyperplane central sections necessarily have smaller volumes. The answer is known to be affirmative if and only if  $n < 5$ . The same question for equilibrated convex bodies in the  $n$ -dimensional complex space  $C^n$  has an affirmative answer if and only if  $n < 4$ . We show that the similar problem in the  $n$ -dimensional quaternionic space  $H^n$  has an affirmative answer if and only if  $n = 2$ . Our method relies on the properties of Radon and cosine transforms on the unit sphere and provides alternative proofs to real and complex cases.

**Michael Ruzhansky**

Imperial College London, UK

*e-mail: m.ruzhansky@imperial.ac.uk*

**Smoothing estimates for evolution equations**

We will present an approach to the smoothing estimates for evolution equations based on the canonical transforms and comparison principles for evolution PDEs. In particular, we will show that most of the smoothing estimates for different a-priori unrelated equations with constant coefficients are equivalent to each other. We will also discuss global estimates for Fourier integral operators that we establish for such purpose.

**Vladimir Ryazanov**

National Academy of Sciences of Ukraine, Ukraine

*e-mail: vlryazanov1@rambler.ru*

**BMO and Dirichlet problem for Beltrami equations**

(joint work with Yury Dybov)

Let  $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ . The equation

$$f_{\bar{z}} = \mu(z) f_z, \tag{2}$$

where  $z = x + iy$ ,  $f_{\bar{z}} = (f_x + if_y)/2$ ,  $f_z = (f_x - if_y)/2$  and  $|\mu(z)| < 1$  a.e., is said to be a **Beltrami equation**. The function  $\mu(z) = f_{\bar{z}}/f_z$  is said to be a **complex dilatation** of  $f$  and the function

$$K_\mu(z) = \frac{1 + |\mu(z)|}{1 - |\mu(z)|} \quad (3)$$

a **maximal dilatation** of  $f$ . The equation (2) is said to be **degenerate** if  $K_\mu \notin L^\infty(D)$ , i.e. if there is no constant  $K \in [1, \infty)$  such that  $K_\mu(z) \leq K$  a.e. Let  $\mathbb{D}$  be the unit disk  $\{z \in \mathbb{C} : |z| < 1\}$ .

Given a continuous function  $\varphi : \partial\mathbb{D} \rightarrow \mathbb{R}$ , we consider a **Dirichlet problem** for the equation (2) consisting in finding a continuous function  $f : \mathbb{D} \rightarrow \mathbb{C}$  having a partial derivatives in  $\mathbb{D}$  and satisfying the following boundary condition for (2):

$$\lim_{z \rightarrow \zeta} \operatorname{Re} f(z) = \varphi(\zeta) \quad \forall \zeta \in \partial\mathbb{D}. \quad (4)$$

Under  $\varphi(\zeta) \neq \text{const}$ , a discrete open function  $f : \mathbb{D} \rightarrow \mathbb{C}$  of the class  $W_{loc}^{1,1}$  is called a **regular solution** of the Dirichlet problem (4) for (2) if  $f$  satisfies (2) a.e. and (4) and its Jacobian  $J_f(z) = |f_z|^2 - |f_{\bar{z}}|^2 \neq 0$  a.e. and  $\operatorname{Im} f(0) = 0$ . Under  $\varphi(\zeta) \equiv c$ ,  $\zeta \in \partial\mathbb{D}$ , the regular solution is  $f(z) \equiv c$ ,  $z \in \mathbb{D}$ .

**Theorem 1.** *Let  $\mu : \mathbb{D} \rightarrow \mathbb{D}$  be a measurable function such that*

$$K_\mu(z) \leq Q(z) \in BMO. \quad (5)$$

*Then the Beltrami equation (2) has a regular solution of the Dirichlet problem (4) for every continuous function  $\varphi : \partial\mathbb{D} \rightarrow \mathbb{R}$ .*

**Emil Saucan**

The Technion - Israel Institute of Technology, Israel

*e-mail: semil@tx.technion.ac.il*

### **Triangulations, quasiregular mappings and differential geometry**

Starting from the existence problem for quasimeromorphic mappings on manifolds, we explore the application of our previous results in this direction and of the methods employed, to the problem of convergence of curvature measures for piecewise at manifolds. Applications are also considered, in particular to Regge Calculus. In addition, we further investigate the connection between quasiregular mappings and curvatures of PL manifolds.

**Fernando Schwartz**

University of Warwick, UK

*e-mail: fernando@math.duke.edu*

**On the topology of black holes**

(3+1)-dimensional black holes can only have cross sections which are topological spheres, but in higher dimensions this is no longer true. In this talk I will elaborate in which cases the preceding statement holds, and show a construction, in terms of Riemannian geometry, that yields cross sections of black holes which are (topologically) a product of spheres.

**Svetlana Selivanova**

Sobolev Institute of Mathematics, Russia

*e-mail: s\_seliv@yahoo.com*

**On some metrical aspects of the theory of Carnot-Carathéodory spaces**

A Carnot-Carathéodory space  $\mathbb{M}$  is a connected Riemannian manifold with a distinguished “horizontal” subbundle  $H\mathbb{M}$  in the tangent bundle  $T\mathbb{M}$ , that meets some algebraic conditions on the commutators of vector fields  $\{X_1, \dots, X_n\}$  constituting a local basis of  $H\mathbb{M}$ ,  $n = n(x) = \dim(H_x\mathbb{M}) \leq \dim(T_x\mathbb{M})$  (versions of the Hörmander’s condition stating that the commutators up to some order span the whole tangent space  $T_x\mathbb{M}$  at any point  $x \in \mathbb{M}$ ). A point  $g \in \mathbb{M}$  is called regular, if there is a neighborhood  $U(g) \subset \mathbb{M}$ , such that  $n(x) = \text{const}$  for all  $x \in U(g)$ . Geometric questions of the theory of Carnot-Carathéodory spaces (also referred to as sub-Riemannian geometry) have many applications in mathematics and physics, in particular they are closely related to the study of fundamental solutions of subelliptic equations  $(\sum_{i=1}^n X_i^2)u = f$  (see e.g. [1]), as well as to optimal control theory for systems

of the form  $\dot{x} = \sum_{i=1}^n u_i(t)X_i(x)$ .

The study of local geometry of  $\mathbb{M}$  heavily relies on the convergence theory for metric spaces developed by M. Gromov, that allows to define the tangent cone to an abstract metric space (which is an analog of the tangent space to a smooth manifold). Usually, the existence of an intrinsic metric  $d_c$  on  $\mathbb{M}$  is derived from the Rashevskii-Chows theorem saying that any two points of  $\mathbb{M}$  can be connected by a “horizontal” curve.

We consider the most general situation [2] (with minimal smoothness  $C^1$  of vector fields  $X_i$  generating  $H\mathbb{M}$  and generalized Hörmander's condition) when the metric  $d_c$  might not exist. But there always exists a quasimetric  $d_\infty$ [2, 3] (it meets the generalized triangle inequality  $d_\infty(u, v) \leq Q(d_\infty(u, w) + d_\infty(w, v))$  for a constant  $1 \leq Q < \infty$ ). For various reasons, it is impossible to straightforwardly generalize Gromov's theory to the case of quasimetric spaces.

In [4] we introduce an adequate for our purposes theory of convergence for quasimetric spaces, the definitions of which are equivalent to the Gromov's ones in the category of metric spaces. Using this theory, we show that the tangent cone to  $(\mathbb{M}, d_\infty)$  at a regular point is a Lie group, the Lie algebra of which is graded and nilpotent (this result is an analog of Mitchell's theorem for  $(\mathbb{M}, d_c)$ ). The proof of this fact is based on the local approximation theorem [2] and is applicable to a more general class of spaces, namely abstract quasimetric spaces with dilations. The study of such spaces is of independent interest.

## References

- [1] L. P. Rotshild, E. M. Stein, Hypoelliptic differential operators and nilpotent groups, *Acta Math.* **137** (1976), 247–320.
- [2] M. Karmanova, S. Vodopyanov, Geometry of Carnot-Carathéodory spaces, differentiability and coarea formula, *Analysis and mathematical physics*, Basel: Birkhäuser (2009), to appear.
- [3] A. Nagel, E. M. Stein and S. Wainger, Balls and metrics defined by vector fields I: basic properties, *Acta Math.* **155** (1985), 103–147.
- [4] S. V. Selivanova, The tangent cone to a quasimetric space with dilations, *Sib. Math. J.* (2009), to appear.

**Avraham Sidi**

The Technion - Israel Institute of Technology, Israel

*e-mail: asidi@cs.technion.ac.il*

**Vector-valued rational interpolation in the complex plane**

We construct several vector-valued rational interpolation procedures that can be implemented easily. We then study the convergence of one of these procedures in the context of meromorphic functions in the complex plane. In particular, choosing the points of interpolation appropriately, we prove a de Montessus type convergence theorem for the interpolants and Koenig type theorems for their poles.

**Brian Smith**

Freie Universität Berlin, Germany

*e-mail: bsmith@math.fu-berlin.de*

**Black hole initial data with a horizon of prescribed intrinsic geometry**

The purpose of this talk is to present a method for constructing asymptotically flat, time symmetric initial data with a horizon of prescribed geometry. The main tool employed is the parabolic partial differential equation for prescribing scalar curvature, for which the horizon geometry is contained within the freely specifiable part of the metric. This contrasts with the conformal method in which the geometry of the horizon can only be specified up to a conformal factor.

**Jacques Smulevici**

Cambridge University, UK

*e-mail: j.smulevici@damtp.cam.ac.uk*

**Structure of singularities of spacetimes with toroidal or hyperbolic symmetry**

I will present recent results concerning the study of the global dynamics in general relativity under symmetry assumptions. More specifically, I will be focusing on the structure of singularities and the uniqueness in the large for solutions of the Einstein equations, the so-called strong cosmic censorship, under the assumption that the initial data is given on some compact manifold

with prescribed symmetry. In particular, I will present some results which concerned the asymptotic behaviour of the area of the orbits of symmetry, a quantity which plays an important role in the study of these solutions. From the point of view of PDE, these results correspond to global existence theorems for certain systems of non-linear wave equations in 1+1 dimensions.

**Alex Solynin**

Texas Tech University, USA

*e-mail: alex.solynin@ttu.edu*

**Some recent results in classical complex analysis**

I will discuss several questions raised in recent publications of mathematicians working in classical complex analysis and related areas. In particular, I will mention the following:

- (1) A new variant of the Schwarz lemma and its application to the hyperbolic metric.
- (2) D. Marshall's problem on the "containment radius" for analytic functions.
- (3) Problem on the hyperbolic convexity of the analytic fixed point function posed by D. Mejia and Ch. Pommerenke in 2005 in connection with their work on applications of complex analysis to probability theory.

**Ilya Spitkovsky**

College of William and Mary, USA

*e-mail: ilya@math.wm.edu*

**Factorization of almost periodic matrix functions, and related functional equations**

Factorization of almost periodic matrix functions is a challenging problem. Constructive criteria of its existence, even in the 2-by-2 triangular case (arising naturally in applications to convolution type equations on finite intervals) are still unknown. We will discuss some recent developments in this area, involving matrices associated with certain difference equations on the unit circle, as well as further results based on the Portuguese transformation.

**Ted J. Suffridge**

University of Kentucky, USA

*e-mail: ted@ms.uky.edu*

**De la Vallée Poussin Means of convex holomorphic mappings of the ball in  $\mathbb{C}^n$**   
(joint work with R. Muir)

Following some ideas of Pólya and Schoenberg, in a 1958 paper, we define de la Vallée Poussin means of holomorphic mappings of the ball in complex  $n$ -space. We prove some theorems concerning subordination and geometric properties of the means (polynomials), and state some conjectures.

In a 1958 paper, Pólya and Schoenberg studied the de la Vallée Poussin kernels

$$\omega_n(t) = \frac{2^n (n!)^2}{(2n)!} (1 + \cos(t))^n = \frac{1}{\binom{2n}{n}} \sum_{k=-n}^n \binom{2n}{n+k} e^{ikt}.$$

For a function  $f$  that is integrable on the interval  $[0, 2\pi]$  they proved some variation diminishing properties of the de la Vallée Poussin means

$$v_n(f, t) = \frac{1}{2\pi} \int_0^{2\pi} \omega_n(t - \tau) f(\tau) d\tau, \quad n \in \mathbb{N}.$$

Further, they were able to interpret their results for analytic functions and proved, for example, that if  $f(z) = \sum_{k=1}^{\infty} a_k z^k$  is analytic on the unit disk and maps the unit disk onto a convex domain, then, for each  $n \in \mathbb{N}$ , the function (a polynomial of degree  $\leq n$ )

$$V_n(f, z) = \frac{1}{2\pi} \int_0^{2\pi} \omega_n(-\tau) f(ze^{i\tau}) d\tau = \sum_{k=1}^n \frac{(n!)^2}{(n-k)!(n+k)!} a_k z^k$$

also maps the unit disk onto a convex domain and  $V_n(f, z) \prec f$ . A number of conjectures were then stated and those were eventually proved.

In this work, we consider holomorphic mappings  $f : B \rightarrow \mathbb{C}^m$  given by  $f(z) = \sum_{k=1}^{\infty} P_k(z)$  where  $B$  is the Euclidean ball in  $\mathbb{C}^m$  and  $P_k$  is a homogeneous polynomial of degree  $k$ . The de la Vallée Poussin means of  $f$  are then given by

$$V_n(f, z) = \frac{1}{2\pi} \int_0^{2\pi} \omega_n(-\tau) f(ze^{i\tau}) d\tau = \sum_{k=1}^n \frac{(n!)^2}{(n-k)!(n+k)!} P_k(z).$$

In this case, we show that  $V_n(f, z) \prec f$ , and that if  $f$  is a holomorphic mapping on  $B$  and  $V_n(f, z) \prec f$  for all large  $n$ , then  $f$  has the property that

$$\operatorname{Re}\langle Df(z)^{-1}(D^2f(z)(z, z) + Df(z)(z)), z \rangle > 0.$$

This is the necessary and sufficient condition for convexity for  $m = 1$ , it is necessary but not sufficient when  $m \geq 2$ . Further, we discuss the question of whether a holomorphic mapping,  $f$ , that is bounded with  $f(B)$  convex can be extended to be continuous on the closed ball, and the relation of this question to the means  $V_n(f, z)$ . In addition, we consider the continuous extension of these means,  $V_\lambda$ ,  $\lambda > 0$  defined by Ruscheweyh and Suffridge.

**Nikolai Tarkhanov**

University of Potsdam, Germany

*e-mail: tarkhanovs@web.de*

#### **An explicit Carleman formula for the Dolbeault cohomology**

We study formulas which recover a Dolbeault cohomology class in a domain of  $\mathbb{C}^n$  through its values on an open part of the boundary. These are called Carleman formulas after the mathematician who first used such a formula for a simple problem of analytic continuation. For functions of several complex variables our approach gives the simplest formula of analytic continuation from a part of the boundary. The extension problem for the Dolbeault cohomology proves surprisingly to be stable at positive steps if the data are given on a concave piece of the boundary. In this case we construct an explicit extension formula.

**Vladimir Turetsky**

The Technion - Israel Institute of Technology, Israel

*e-mail: turetsky@aerodyne.technion.ac.il*

#### **Cheap control in generalized linear-quadratic differential game**

(joint work with Valery Y. Glizer)

In this talk, a linear-quadratic differential game of general type is considered. In this game, the state term of the cost functional is a Lebesgue-Stieltjes integral of squared discrepancy between the system motion and the given vector function, calculated over the mixed discrete/continuous measure. The

control term of the cost functional represents the penalties for the control and the disturbance expenditure. The game is solved, based on a program maximin calculation. The solution exists if the maximal eigenvalue of some integral operator in the Hilbert functional space is less than one. The game value is given by a quadratic form in this space. The respective impulsive differential equations (one of Riccati type and one linear) for the coefficients of the optimal linear feedbacks are derived. A cheap control version of the game, arising in aerospace applications, is analyzed. It is shown that if the penalty coefficients tend to zero in concord with each other then (i) the game value tends to zero; (ii) the state term of the cost functional also tends to zero; (iii) the control realization remains bounded. An example of aerospace application is presented.

**Alexander Ukhlov**

Ben-Gurion University of the Negev, Israel  
*e-mail: ukhlov@math.bgu.ac.il*

### **Composition Operators in Sobolev spaces**

We study compositions operator in Sobolev spaces for homeomorphisms of Euclidean domains. These operators are closely connected with generalized quasiconformal mappings. Mappings of such type have applications to the nonlinear elasticity theory and to the Sobolev type embedding theory.

**Alexander Vasiliev**

University of Bergen, Norway  
*e-mail: ava004@math.uib.no*

### **Conformal field theory viewpoint on contour dynamics**

We shall discuss relations between classical and stochastic (Schramm) Loewner equations and conformal field theory. In particular, the Witt and Virasoro algebras play a structural role there. We also give some connections with sub-Riemannian geometry.

**Fabio Vlacci**

Università di Firenze, Italy

*e-mail: vlacci@math.unifi.it*

**Rigidity for holomorphic and regular maps**

Some (boundary) rigidity results for holomorphic maps have been recently extended for the case of quaternionic regular functions but some other remain an open question; the aim of this talk is to show the analogies (or differences) of the approach to the problem in the two settings.

**Sergey Vodopyanov**

Sobolev Institute of Mathematics, Russia

*e-mail: vodopis@math.nsc.ru*

**Differential forms and mappings with controlled distortion**

We investigate necessary and sufficient conditions on approximately differentiable mappings  $f : \mathbb{M} \rightarrow \mathbb{M}'$  of Riemannian manifolds to induce a bounded (with respect to Lebesgue spaces norms) pull-back operator of differential forms. As a consequence, we obtain, in particular, that a homeomorphism  $f : \mathbb{M} \rightarrow \mathbb{M}'$  of the class  $\text{ACL}(\mathbb{M})$ , for which the pull-back operator of differential forms with the norm in  $\mathcal{L}_p$  is an isomorphism, is either quasi-conformal or quasiisometric.

**Yitzhak Weit**

University of Haifa, Israel

*e-mail: weit@math.haifa.ac.il*

**On the translates of powers of a continuous periodic function**

We characterize the set of real-valued,  $2\pi$ -periodic, continuous functions  $f$  for which the translation invariant subspace  $V(f)$  generated by  $f^n$ ,  $n \geq 0$ , is dense in  $C(\mathbb{T})$ . In particular, it follows that if  $f$  takes a given value at only one point then  $V(f)$  is dense in  $C(\mathbb{T})$ .

**Catherine Williams**  
Stanford University, USA  
*e-mail: cathwill@stanford.edu*

**Predicting long term behavior of marginally trapped tubes from initial data**

Marginally trapped tubes (MTTs) are spacetimehypersurfaces with an array of interesting mathematical properties; in physical contexts, such objects are sometimes interpreted as quasi-local black hole boundaries. This talk will focus on the asymptotic behavior of MTTs in relation to traditional black hole event horizons. In particular, for spherically symmetric black hole spacetimes whose matter is determined by a self-gravitating nonlinear scalar field, it turns out that certain smallness and mild decay assumptions on asymptotically flat initial data are sufficient to guarantee the subsequent ‘nice’ causal and long term behavior of the MTT inside the black hole. This result can be loosely interpreted as a statement about the stability of ‘nice’ asymptotic behavior of MTTs under certain small perturbations of Schwarzschild.

**Jens Wirth**  
Imperial College London, UK  
*e-mail: jwirth@imperial.ac.uk*

**Decay estimates for anisotropic thermo-elasticity**

The equations of thermo-elasticity couple a hyperbolic system (the system of crystal acoustics) to a heat equation. Aim of the talk is to present (sharp and frequency-localised) a priori estimates for solutions in terms of geometric properties of the symbol of the elastic operator. The approach is based on an asymptotic de-coupling of the system for small and large frequencies. Several examples for the two-dimensional situation will be given and the particular situation of cubic media in three space dimensions discussed in some detail. The two-dimensional treatment is based on joint work with Michael Reissig (Freiberg).

**Vyacheslav Zaharyuta**  
Sabancı University, Turkey  
*e-mail: zaha@sabanciuniv.edu*

### **Bases in spaces of analytic functions and applications**

$A(D)$  is the space of all functions analytic in an open subset  $D$  in a Stein manifold  $\Omega$  with the topology of locally uniform convergence in  $D$ . For an arbitrary set  $E \subset \Omega$ ,  $A(E)$  is a set of all analytic germs on  $E$  considered with the locally convex topology of the inductive limit  $A(E) = \lim_{G \in \mathcal{O}(E)} A(G)$ , where  $\mathcal{O}(E)$  is the set of all open neighborhoods of  $E$ .

We are going to discuss the following topics:

- Existence of bases in those spaces
- Their construction and structure
- Common basis for a pluriregular condenser
- Isomorphic classification of spaces  $A(D)$
- Applications to approximation and analytic extension.

**Mikhail Zaidenberg**  
Institut Fourier, France  
*e-mail: Mikhail.Zaidenberg@ujf-grenoble.fr*

### **Playing “Lights out”, or Harmonic functions on lattices and points count in positive characteristic**

The game “Lights out”, commercialized by “Tiger Electronics”, occurred to be related to simplest cellular automata. It became a source of inspiration for the work of Sutner, Goldwasser-Klostermayer-Ward, Barua-Sarkar, Hunziker-Machiavello-Park e.a.

Suppose that (by a security reason) the interrupters in your department are synchronized in such a way that switching off in one office automatically changes the state to the opposite in all neighbor offices separated by a wall. How can the person leaving the department the last turn all the lights off?

Numerous problems arise and mainly remain open. They can be formulated as spectral problems, in terms of harmonic analysis, or as problems about Chebyshev polynomials in characteristic 2. Alternatively, they lead to unusual but intriguing questions in arithmetic algebraic geometry, concerning torsion points of certain elliptic curves or more general affine algebraic varieties. We will present some computer simulations done by Zagier, and some recent progress related this to Artin's problem on primitive roots.

**Jozef Zajac**

Catholic University of Lublin & State University of Applied Science in Chelm,  
Poland

*e-mail: jzajac@kul.lublin.pl*

**Harmonic mappings and quasihomographies in the theory of  
Teichmüller space**

One of the most powerful tools, when studying Riemann surface, is the notation of Teichmüller space, i.e. a metrizable and complete quotient space of closed Riemann surfaces with genus  $g > 2$ . While the concept was introduced by ingenious German mathematician O. Teichmüller before World War II, the name appears because of L. Bers and L. V. Ahlfors in the late fifties. The function theoretic model of this, not easy understandable, original Teichmüller space, was built up by the use of equivalence classes of quasiconformal automorphisms of the unit disc or its boundary representation called quasihomographies. Making use of the Poisson integral extension operator one may construct harmonic representation of the universal Teichmüller space in which, particular, boundary normalized harmonic automorphisms of the unit disc, represent elements of the space, in question.

The main purpose of the lecture is to present a number of theorems and constructions regarding metric and topological feature of harmonic and quasihomographic models of the universal Teichmüller space. It is worth to mention that the harmonic representation of the universal Teichmüller space was designed by E. Paprocki a young mathematician killed before his, high estimated doctorate, in a road accident. His idea links once again extremal quasiconformal automorphisms of the unit disc with two classes of analytic functions, defined in the unit disc called the conjugate Paprocki spaces of analytic functions. Some basic properties of functions from those spaces will be also presented during this lecture.

**Alexander Zaslavski**

The Technion - Israel Institute of Technology, Israel

*e-mail: ajzasl@tx.technion.ac.il*

**Optimal control systems arising in economic dynamics**

We discuss the structure of approximate solutions of an autonomous discrete-time control system with a compact metric space of states  $X$ . This control system is described by a bounded upper semicontinuous function  $v : X \times X \rightarrow R^1$  which determines an optimality criterion and by a nonempty closed set  $\Omega \subset X \times X$  which determines a class of admissible trajectories (programs). We are interested in turnpike properties of the approximate solutions which are independent of the length of the interval, for all sufficiently large intervals.

**Yuri Zelinskii**

National Academy of Sciences of Ukraine, Ukraine

*e-mail: zel@imath.kiev.ua*

**Continuous mappings between domains of manifolds**

Let  $f$  be continuous mapping between domains of manifolds, with disjoint images of the boundary and of the domain interior, and certain degree  $k$ , then either the mapping is interior in the sense of Stoilow or there is a point possessing at least  $|k| + 2$  preimages. If additionally in the last case the map  $f$  be zerodimensional in the domain interior, then the set of points possessing at least  $|k| + 2$  preimages contains open ball.

**Jaroslav Zemanek**

Institute of Mathematics of the Polish Academy of Sciences, Poland

*e-mail: zemanek@impan.pl*

**On operators with single spectrum**

We intend to discuss recent results and questions concerning analytic and geometric properties of linear operators with the single spectrum  $\{1\}$ .

**Nahum Zobin**

College of William and Mary, USA

*e-mail: nxzobi@wm.edu*

**Symbols of operators, Fourier transforms on Lie groups and functional integrals**

For a Lie group one can define a quantization mapping as a composition of a usual inverse Fourier transform on the dual space to the Lie algebra, the exponential mapping and the Fourier transform on the group. The usual Weyl quantization is related to the simplest non-commutative group – the Weyl-Heisenberg group. We explain this procedure for a general Lie group, its relation to the Orbit Method, and apply it to an operator calculus, expressed in terms of a functional integral over the space of probability measures on the dual space of the Lie algebra. These considerations are mostly motivated by quantum gauge field theories on a non-commutative space-time, which require the quantization procedure for the group  $SO(1, 4)$ .